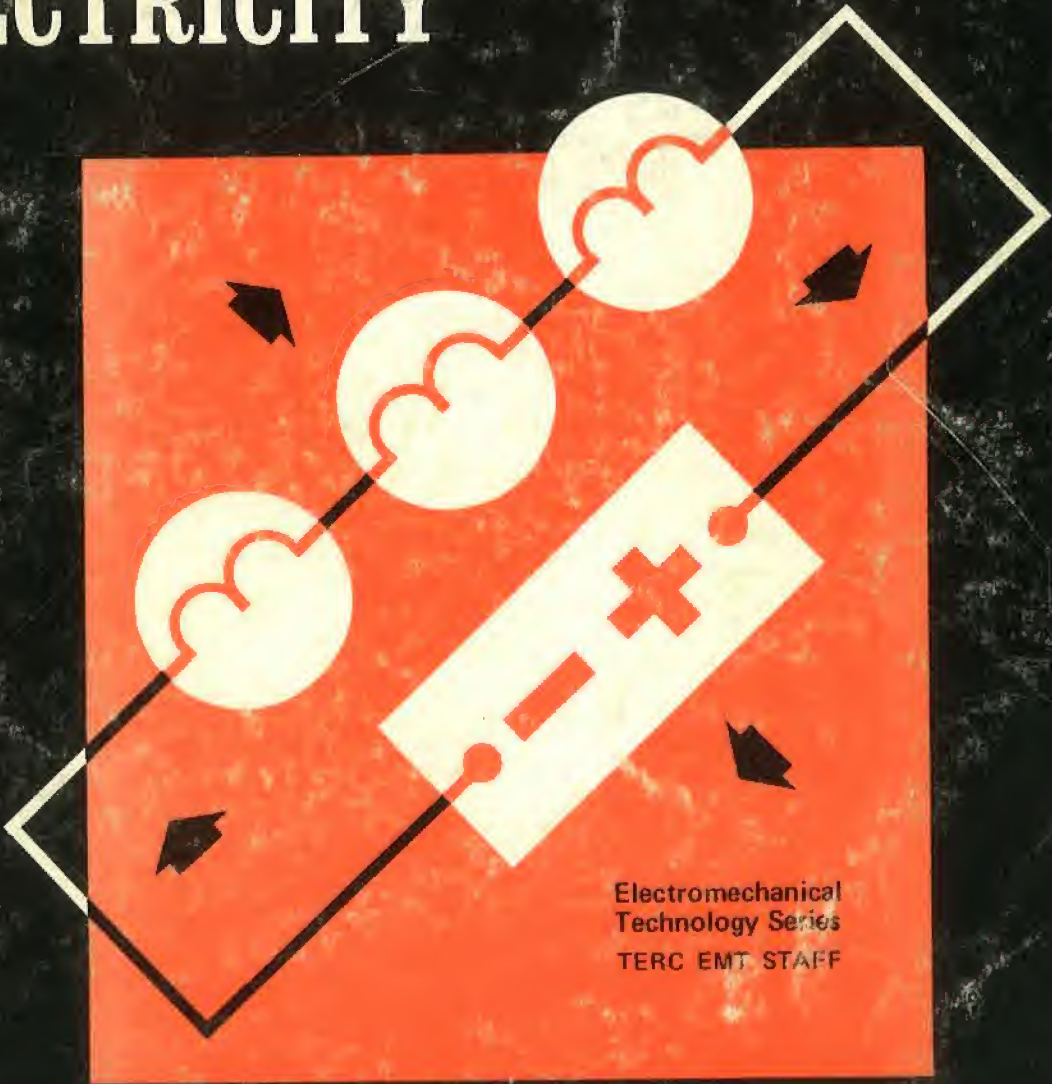
TERC

ELECTRONICS

ELECTRICITY



Electromechanical
Technology Series

TERC EMT STAFF

DELMAR PUBLISHERS, MOUNTAINVIEW AVENUE, ALBANY, NEW YORK 12205

ELECTRONICS

ELECTRICITY

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Foreword

The marriage of electronics and technology is creating new demands for technical personnel in today's industries. New occupations have emerged with combination skill requirements well beyond the capability of many technical specialists. Increasingly, technicians who work with systems and devices of many kinds — mechanical, hydraulic, pneumatic, thermal, and optical — must be competent also in electronics. This need for combination skills is especially significant for the youngster who is preparing for a career in industrial technology.

This manual is one of a series of closely related publications designed for students who want the broadest possible introduction to technical occupations. The most effective use of these manuals is as combination textbook-laboratory guides for a full-time, post-secondary school study program that provides parallel and concurrent courses in electronics, mechanics, physics, mathematics, technical writing, and electromechanical applications.

A unique feature of the manuals in this series is the close correlation of technical laboratory study with mathematics and physics concepts. Each topic is studied by use of practical examples using modern industrial applications. The reinforcement obtained from multiple applications of the concepts has been shown to be extremely effective, especially for students with widely diverse educational backgrounds. Experience has shown that typical junior college or technical school students can make satisfactory progress in a well-coordinated program using these manuals as the primary instructional material.

School administrators will be interested in the potential of these manuals to support a common first-year core of studies for two-year programs in such fields as: instrumentation, automation, mechanical design, or quality assurance. This form of *technical core* program has the advantage of reducing instructional costs without the corresponding decrease in holding power so frequently found in general core programs.

This manual, along with the others in the series, is the result of six years of research and development by the *Technical Education Research Center, Inc.,* (TERC), a national nonprofit, public service corporation with headquarters in Cambridge, Massachusetts. It has undergone a number of revisions as a direct result of experience gained with students in technical schools and community colleges throughout the country.

Maurice W. Roney

The Electromechanical Series

TERC is engaged in an on-going educational program in *Electromechanical Technology.* The following titles have been developed for this program:

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Preface

Technology, by its very nature, is a laboratory-oriented activity. As such, the laboratory portion of any technology program is vitally important. *Electronics/Electricity* is intended to provide meaningful experience in electric circuit analysis for students of modern technology.

The topics included provide exposure to basic principles of current flow, simple DC circuits, an introduction to AC circuits, resonance, and transformer coupling.

The sequence of presentation chosen is by no means inflexible. It is expected that individual instructors may choose to use the materials in other than the given sequence.

The particular topics chosen for inclusion in this volume were selected primarily for convenience and economy of materials. Some instructors may wish to omit some of the exercises or to supplement some of them to better meet their local needs.

The materials are presented in an action-oriented format combining many of the features normally found in a textbook with those usually associated with a laboratory manual. Each experiment contains:

1. An INTRODUCTION which identifies the topic to be examined and often includes a rationale for doing the exercise.
2. A DISCUSSION which presents the background, theory, or techniques needed to carry out the exercise.
3. A MATERIALS list which identifies all of the items needed in the laboratory experiment. (Items usually supplied by the student such as pencil and paper are not included in the lists.)
4. A PROCEDURE which presents step-by-step instructions for performing the experiment. In most instances the measurements are done before calculations so that all of the students can at least finish making the measurements before the laboratory period ends.
5. An ANALYSIS GUIDE which offers suggestions as to how the student might approach interpretation of the data in order to draw conclusions from it.
6. PROBLEMS are included for the purpose of reviewing and reinforcing the points covered in the exercise. The problems may be of the numerical solution type or simply questions about the exercise.

Laboratory report writing forms an important part of the learning process included in this manual. To serve as a guide in this activity, instructions for preparing reports as well as sample reports are included for student use.

Students should be encouraged to study the textual material, perform the experiment, work the review problems, and submit a technical report on each topic. Following this pattern, the student can acquire an understanding of, and skill with, basic electric circuits that will be extremely valuable on the job. For best results, these students should be concurrently enrolled in a course in technical mathematics (algebra and trigonometry).

These materials on basic electricity comprise one of a series of volumes prepared for technical students by the TERC EMT staff at Oklahoma State University, under the direction of D.S. Phillips and R.W. Tinnell. The principal author of these materials was R.W. Tinnell.

An *Instructor's Data Guide* is available for use with this volume. Mr. Kenneth F. Cathy was responsible for testing the materials and compiling the instructor's data book for them. Other members of the TERC staff made valuable contributions in the form of criticisms, corrections, and suggestions.

It is sincerely hoped that this volume as well as the other volumes in this series, the instructor's data books, and other supplementary materials will make the study of technology interesting and rewarding for both students and teachers.

THE TERC EMT STAFF

TO THE STUDENT

Duplicate data sheets for each experiment are provided in the back of the book. These are perforated to be removed and completed while performing each experiment. They may then be submitted with the experiment analysis for your instructor's examination.

Contents

experiment 1 METER CONNECTIONS

INTRODUCTION. This laboratory course, like many others, is intended to reinforce, extend, and deepen your understanding of electrical principles. The success of the course depends greatly on your attitude and performance in it.

This first experiment will introduce some of the basic techniques to be used throughout the course. In particular, the connection and reading of basic meters will be considered.

DISCUSSION. *Meters* are perhaps the basic measuring devices encountered in electrical laboratories. The three quantities most frequently measured with an ordinary meter are electrical pressure *(voltage)*, electrical flow *(current)*, and the opposition to flow *(resistance)*. Let us consider each of these measurements individually.

Electrical pressure or voltage may be measured in *microvolts* (μV), *millivolts* (mV), *volts* (V), or *kilovolts* (kV); but in any case, the technique is substantially the same.

Voltage, being a type of pressure, is measured in the same way as any other pressure. That is, we connect the meter directly between the two points whose pressure difference we wish to measure. As an example, let us suppose that we wish to measure the voltage at the terminals of a *battery*. To do so, we attach the lead wires of the meter directly to the two battery terminals, see figure 1-1.

While the basic technique for measuring voltage is very simple, there are a number of details which tend to complicate it somewhat. A satisfactory procedure for most instruments is as follows:

1. Many instruments have a *function selector* switch. This switch will select between a number of functions such as: DC voltage, AC voltage, DC current, ohms, etc. This switch should be set at the appropriate position (in this case, DC voltage).
2. The instrument will frequently have a *range selector* switch to choose the operating range of the meter. Typical markings for a range selector are 1000V, 250V, 10V, 2.5V, etc. The range selector should be set to the range which just exceeds the voltage to be measured. If there is any doubt about the size of the voltage to be measured, the range selector should be set for the highest range (1000V in this case).

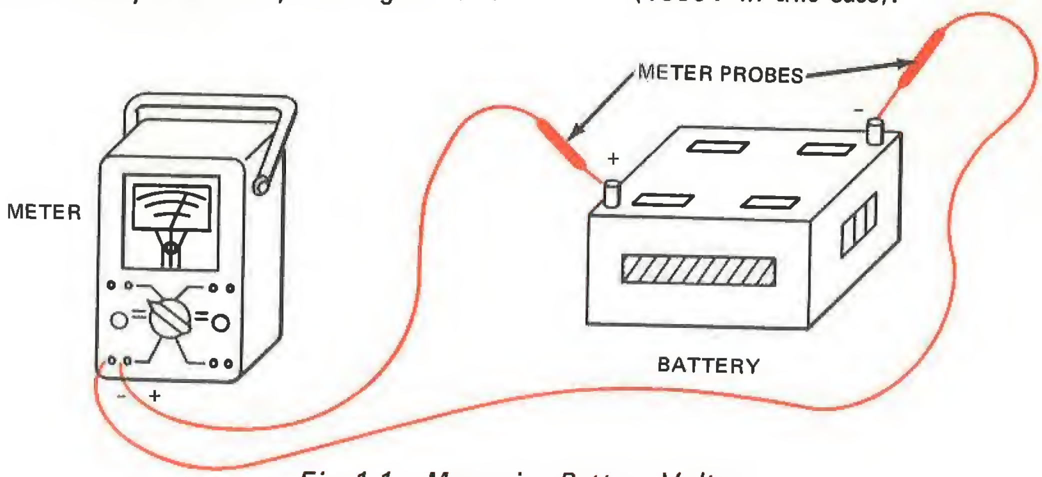


Fig. 1-1 Measuring Battery Voltage

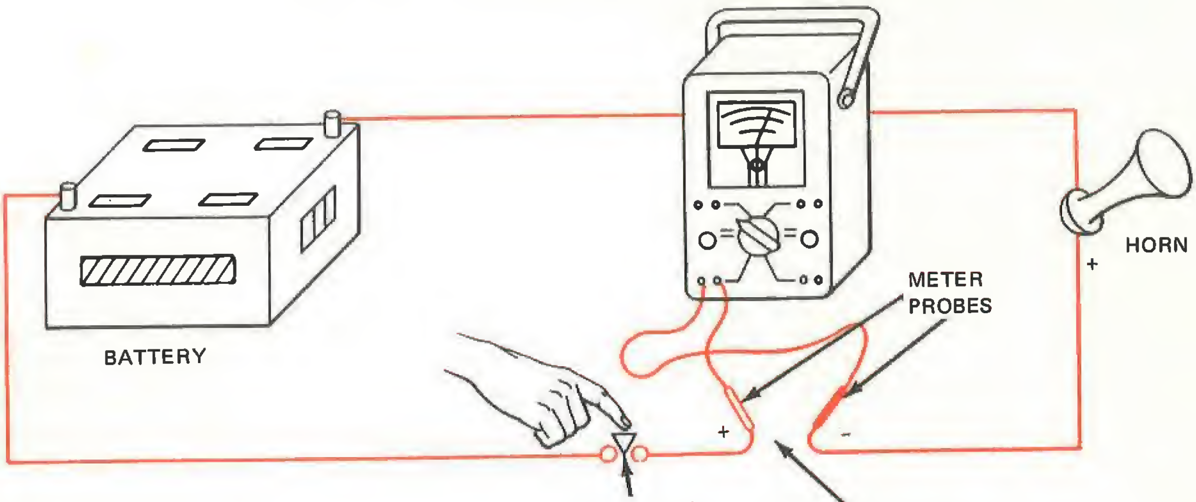


Fig. 1-2 Measuring Current Flow Through a Horn

3. Virtually all meters have a *mechanical zero adjustment* screw. This screw will be located directly below the meter face in the center of the instrument. It should be adjusted so that the meter pointer indicates zero with the meter disconnected and in position for normal operation.
4. Voltmeters are sensitive to the polarity of the voltage being measured; therefore, some care must be taken to connect the positive meter lead to the positive voltage terminal and the negative meter lead to the negative terminal.
5. When the meter is connected, the range selector position is moved to a lower range, if necessary, to get a readable indication.
6. Many meters have several scales marked on their face. Some care is required to insure that the reading is taken from the scale which corresponds to the settings of both the function and range selectors.

Electrical current meters measure the rate of flow of electricity. As such, a current meter must be inserted into the electrical circuit in a manner which allows the current to flow through the instrument. In using a current meter, very much the same procedure as used with a voltmeter may be employed. In summary form the procedure is:

1. Set the function selector to DC current.
2. Set the range selector to the appropriate range or a higher one.
3. Check the mechanical zero adjustment.
4. Observe polarity requirements.
5. Insert the meter and correct the range selection if necessary.
6. Read the appropriate scale.

It should be emphasized at this point that a current meter will almost certainly be damaged if it is connected as a voltmeter. Current meters *must be inserted* into the current path as shown in figure 1-2.

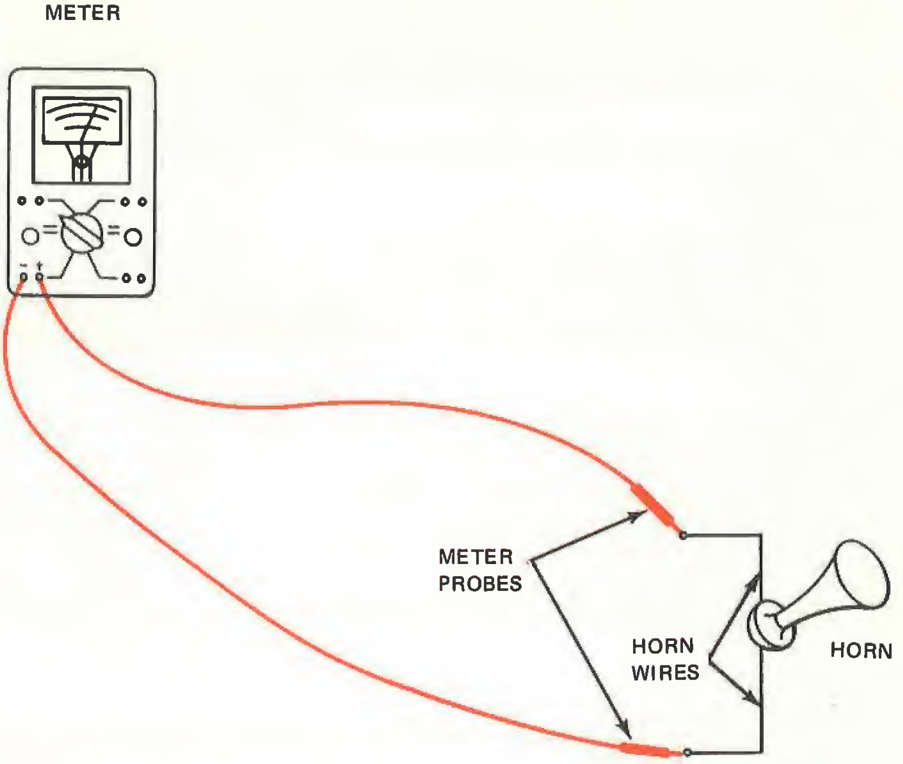


Fig. 1-3 Measuring Horn Resistance

In using a meter to measure the resistance (opposition to current flow) of an electrical device, one uses many of the same techniques used in measuring voltage and current. Indeed, the initial steps in preparing to measure resistance are identical to those described previously. That is:

1. Set the function selector to the DC voltage position. (Some meters have a special *ohms* position which is used instead.)
2. Set range selector to the RxI position.
3. Check the mechanical zero adjustment.

From this point on, the procedure varies from the one discussed previously as follows:

4. Hold the meter lead wires together so that the probes are touching. The meter pointer should swing to the right side of the scale. Adjust the *zero ohms* control until the pointer reads zero on the right side of the ohms scale.
5. Separate the meter leads and set the range selector to the desired range.
6. Disconnect the device whose resistance is to be measured from any other circuit, and connect the meter leads directly across the terminals of the device.
7. Read the resistance of the device on the ohms scale in units of ohms, kilohms, or megohms as determined by the range selector setting.

A sketch illustrating the connection to be used in measuring resistance is shown in figure 1-3.

The three techniques described previously should be adequate to handle most basic electric laboratory experiments.

MATERIALS

1 variable DC power supply, 0-30V

1 multimeter

1 28V pilot lamp (G.E. 313 or equiv.)

1 base for pilot lamp

PROCEDURE

1. Using the multimeter, measure the resistance of the pilot lamp in its base. Record the value of your reading in the data table, figure 1-5, under R_L.
2. Connect the circuit indicated in figure 1-4. DO NOT TURN THE POWER SUPPLY ON

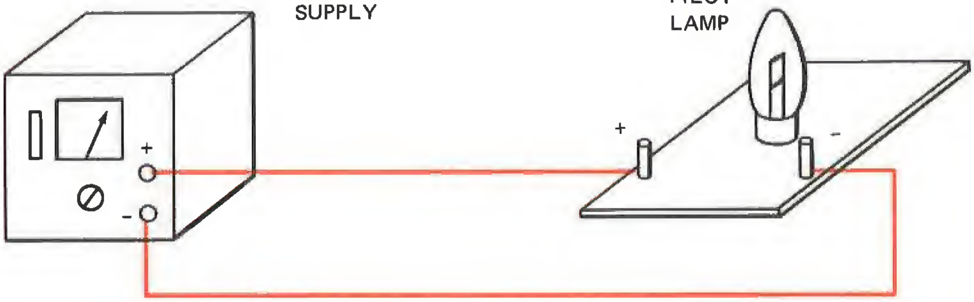


Fig. 1-4 The Experimental Circuit

3. Lay the multimeter flat on the bench so that the meter face is horizontal. Connect the meter leads across the lamp terminals.
4. Have the laboratory instructor check your experiment setup. The instructor will initial your data table if the setup is correct.
5. Turn on the power supply and adjust the voltage to about 9V until the lamp filament is a bright orange color (not white).
6. Record the value of the voltage across the lamp in the data table as E_1. (E_2 in step 13)
7. Disconnect the positive lamp wire and insert the meter for current measurement. *Leave the range and function selectors in the voltage measurement positions.*
8. Have the laboratory instructor check your setup and initial your data table a second time.
9. Set the range and function selectors to the highest current range. Reduce the range selector one step at a time until the current value can be read. Record the value of the current as I_1 in the data table. (This is I_2 in step 13.)
10. Remove the meter and reconnect the circuit as in figure 1-4.
11. Set the function and range selectors for voltage measurement. Measure and record the voltage at the power supply terminals. Record this voltage as E'_1. (E'_2 in step 13)
12. Set the meter upright so that the scale is vertical. *Do not* readjust the mechanical zero adjustment.
13. Repeat steps 6, 7, 9, 10, and 11. Record these data as E_2, I_2, and E'_2 respectively.

14. Return to step 5 and increase the power supply voltage until the lamp filament produces a white colored light. Lay the meter down. (Face horizontal.) Measure and record the lamp voltage. (E_3)
15. Insert the meter in the negative lamp wire. Measure and record the lamp current. (I_3)
16. Remove the meter, reconstruct the circuit, measure and record the power supply voltage. (E'_3)
17. Disconnect the meter and prepare the instrument for measuring the bulb resistance.
18. Disconnect the bulb from the circuit and *quickly* measure and record the bulb resistance. (R'_L)

R_L Ohms	E_1 Volts	I_1 Ma.	E'_1 Volts	E_2 Volts	I_2 Ma.	E'_2 Volts	E_3 Volts	I_3 Ma.	E'_3 Volts	R'_L Ohms

First voltage measurement circuit checked by ______________________________
Instructor

First current measurement circuit checked by ______________________________
Instructor

Fig. 1-5 Data Table

ANALYSIS GUIDE. Since this is the first experiment of the course, no laboratory report will be required. You should, however, answer each of the questions at the end of the experiment and turn in your answers and the data table.

PROBLEMS

1. Did the resistance of the lamp change during the experiment? If so, why do you think it did?
2. Were the values of E_1 and E'_1 different? If so, by how much? Why do you think this occurred? Were E_2 and E'_2 different? E_3 and E'_3?
3. When the lamp was glowing orange, was the current in the positive lamp wire the same as the current in the negative lamp wire? Why do you think your answer is correct?
4. Did the position of the meter (horizontal or vertical) have any effect on the value of the readings? Why is this so?
5. Why was it necessary to make the final bulb resistance reading quickly?

experiment 2 METALLIC CONDUCTORS

INTRODUCTION. *Metallic conductors* play a very important role in all of electricity and electronics. We use them to produce wire and bus bars to convey current from point to point. They are used directly in such components as *coils, lamps, motors,* and *heating* elements as well as providing interconnecting loads for virtually all components. In this experiment, we shall examine some of the important characteristics of these materials.

DISCUSSION. If an *electrical potential* is applied across a metal conductor, the free electrons in the conductor drift toward the positive end. During the drift process, the free electrons proceed from the vicinity of one atom to that of another atom in the metallic structure. At each encounter with a metallic atom, the free electrons give up a small amount of energy in the form of heat. This heat energy must, of course, ultimately be supplied by the potential source.

If a considerable amount of energy is required to cause a given amount of electron current to flow in a conductor, we say that the conductor has a large amount of *resistance* to the flow of current. Conversely, if only a small amount of energy is required for the current flow, we say the conductor has a small amount of resistance.

The number of encounters that a free electron has with the metallic atoms depends directly on the length of the conductor. That is, the longer the conductor, the more opportunities there are for electron-atom interactions. Or, in other words, *the resistance (R) of a conductor is directly related to the length* (ℓ) *of the conductor.*

On the other hand, let us suppose that we take two identical conductors and apply a fixed value of potential across each of them. The total electron drift will be twice as much as the electron drift of one wire alone. Consequently, we can say that the total resistance of the pair of conductors is half that of one conductor alone. Actually, all we have done in adding the second conductor is to double the cross-sectional area of the conducting material available for current flow. Therefore, we may conclude that *the resistance of the conductors is directly related to the inverse of their cross-sectional area.*

Combining this conclusion with the one cited above leads us to a generalization which may be expressed algebraically as

$$R \propto \frac{\ell}{A}$$

In the practical case we almost always work with conductors of circular cross section (wires). For a circular cross section, the area will be

$$A = \frac{\pi d^2}{4}$$

where d is the diameter of the circle and π is equal to about 3.14. However, since the area is directly proportional to the diameter squared, we can rewrite the resistance equation for circular cross-section conductors as

$$\boxed{R = \rho \frac{\ell}{d^2} \text{ ohms}} \qquad (2.1)$$

if we can determine the value of the constant of proportionality (ρ).

This constant (ρ) has been determined experimentally for many different materials and is often called the *resistivity* of the material. The table given in figure 2.1 lists the resistivity of several common conductors. Values for many other materials may be found in physics handbooks and some textbooks.

Material	Resistivity (ohms/mil-foot at 20° C)
Copper	10.4
Aluminum	17
Iron	58
Nickel-Chromium	660

Fig. 2-1 Resistivity of Common Conductors

The term "d^2" is the area of the conductor in circular mils and is quite commonly found in nearly all wire tables.

In employing equation 2-1, the resistivity should be in *ohms per mil-foot,* length should be in *feet,* and the diameter in *mils.* The resistance will then be in *ohms.*

Let us now suppose we wish to determine the resistance of 1000 feet of an iron wire which has a diameter of 100 mils. Using equation 2-1 renders

$$R = \rho \frac{1}{d^2} = 58 \times \frac{1000}{100 \times 100} = 5.8 \text{ ohms}$$

In this experiment, it will be necessary to measure the diameter of several wires with a micrometer. For those not familiar with the use of a *micrometer,* a brief outline for measuring wire diameters follows: (Refer to figure 2-2 for the names of the micrometer parts)

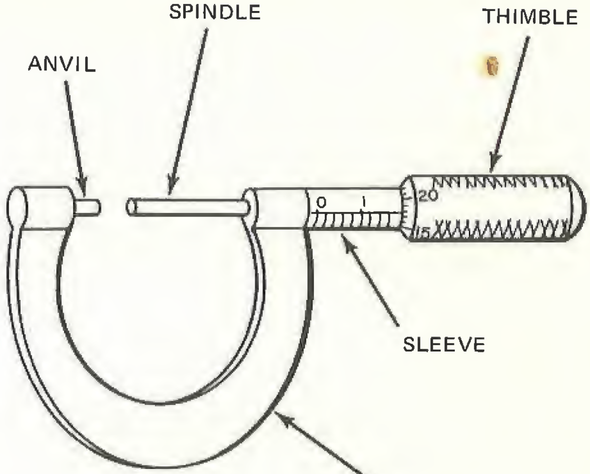


Fig. 2-2 The Micrometer Caliper

1. Place the wire to be measured between the anvil and spindle of the micrometer. Rotate the thimble until the wire *just fits* between the anvil and spindle. Do not squeeze the wire as this will flatten it and give an erroneous diameter measurement.

2. Read the wire diameter on the sleeve and thimble. The sleeve has marks every 25 mils and a digit every 100 mils. The thimble has marks every 1 mil and a number every 5 mils. The reading is made by:

   (a) taking the largest sleeve digit showing times 100 mils (1 x 100 = 100 mils in figure 2-2)

   (b) to this quantity add 25 mils times the number of sleeve marks showing beyond the last digit (100 mils + 25 x 3 = 175 mils in figure 2-2)

   (c) Finally, add the setting of the thimble. (175 mils + 23 = 198 mils in figure 2-2) This is the reading of the micrometer.

MATERIALS

1 Multimeter (VOM)
1 Steel rule, 12 in. long
1 Micrometer caliper (1 inch range)
1 Piece of No. 26 Nichrome wire, 8 in. long
1 Piece of No. 28 Nichrome wire, 8 in. long
5 Pieces of No. 30 Nichrome wire, 4, 6, 8, 10, and 12 in. long
1 Piece of No. 32 Nichrome wire, 8 in. long
1 Piece of No. 34 Nichrome wire, 8 in. long
2 Sheets of linear graph paper, 10 x 10 div. per cm

PROCEDURE

1. Identify the 4-in. piece of No. 30 nichrome wire. Using the steel rule, carefully measure the length of the nichrome section as accurately as possible and record it in the data table (figure 2-4). See figure 2-3 for an illustration of how this measurement is to be made.

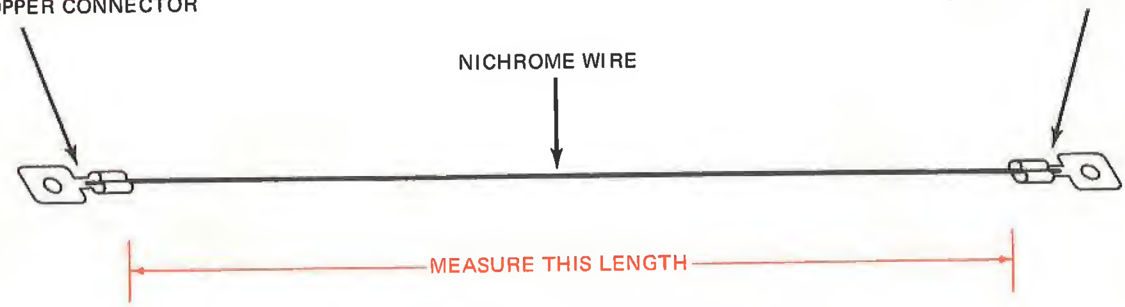


Fig. 2-3 Measurement of Wire Length

2. With the micrometer caliper measure the diameter of the wire at three different locations. Record each measurement.
3. Compute the average value of the three diameter measurements and record it in the data table.
4. Using the multimeter, measure the resistance of the wire and record it in the data table.
5. Assuming that the resistivity of nichrome is the value given in the discussion, compute the value of the resistance of the wire using equation 2.1. Use the measured length and average diameter in this calculation and record the results in the data table.
6. Repeat steps 1 through 5 using the 6-, 8-, 10-, and 12-inch pieces of No. 30 nichrome wire.
7. On a sheet of linear graph paper, plot the measured resistance versus the measured length. The length should be plotted along the horizontal axis and the resistance along the vertical axis.
8. On the same sheet, plot the computed resistance versus the measured length.
9. In the same way, determine the length and average diameter of the 8-inch pieces of No. 26, 28, 32 and 34 nichrome wire. Record all data in the data table.
10. As before, measure and compute the resistance of each piece of wire.
11. Compute the value of d^2 for each of the 8-inch wires. Record this value as the area in circular mils in the data table.

12. Using the equation $A = \frac{\pi d^2}{4}$ compute the cross-sectional area for each of the 8-inch wires. Record this value in the data table.
13. On the second sheet of graph paper, plot the measured resistance versus the area in circular mils.
14. On the same sheet of graph paper, plot the measured resistance versus the cross sectional area.
15. Also plot the computed resistance versus the area in circular mils.

Wire	Length	Dia. (First)	Dia. (Second)	Dia. (Third)	Ave. Dia.	Area Cir. Mils	R Meas.	R Comp	Cross Sect. Area
4 in. No. 30						X			X
6 in. No. 30						X			X
8 in. No. 30									
10 in. No. 30						X			X
12 in. No. 30						X			X
8 in. No. 26									
8 in. No. 28									
8 in. No. 32									
8 in. No. 34									

Fig. 2-4 The Data Table

ANALYSIS GUIDE. In analyzing the data obtained in this experiment, you should first consider whether or not the measured resistance was found to be directly related to the conductor length and inversely related to the cross-sectional area. Also consider the extent to which the computed and measured resistance agreed with one another.

In both cases, use your plots to verify your conclusion.

PROBLEMS

1. What would be the resistance of 35 in. of No. 20 copper wire?
2. A copper bus bar has a rectangular cross section of 0.1 in. by 0.5 in. and is 2 ft. long. What is the resistance of the bus bar? *Hint: Use area in square inch.*
3. How many feet of No. 18 iron wire is required to produce 43 ohms of resistance?
4. A certain conductor has a resistance of 57 ohms. If its length were halved and its cross section doubled, what would be its new resistance?
5. A length of No. 4 copper wire is found to have a resistance of 0.2 ohms. What would be the resistance of an equal length of No. 22 iron wire?
6. What is the difference between a plot of "Resistance versus Area in Circular Mils" and a plot of "Resistance versus Cross-Sectional Area"?

experiment 3 RESISTOR LINEARITY

INTRODUCTION. There are two basic types of resistances in common use in electric circuits. One type is the linear resistor which maintains its resistance at a relatively constant value under normal operating conditions. The other type, the nonlinear resistor, has a resistance value which changes depending on its operating conditions.

In this experiment, we shall examine this property of linearity which separates the two types of resistors.

DISCUSSION. As mentioned above, a linear resistor is a device which has a constant value of resistance under normal circumstances. If the resistance of a device is constant, then the current through it will double if the voltage across it is doubled. This effect was first noted by a 19th century scientist, Georg Simon Ohm, who expressed it algebraically as

$$E = IR \tag{3.1}$$

This relationship has since become known as Ohm's Law and is considered to be one of the foundational concepts of electric theory.

If we plot the current through a linear resistance versus the voltage across it, the result will be similar to one of the lines in figure 3-1. This plot illustrates two important characteristics of linear resistors:

1. *The E-I plot of a linear resistor is always a straight line.*
2. *The slope of the E-I plot is inversely proportional to the ohmic value of the resistor, that is, the steeper the slope of the line, the smaller the resistance.*

The value of a linear resistor can be determined from the E-I plot by choosing any

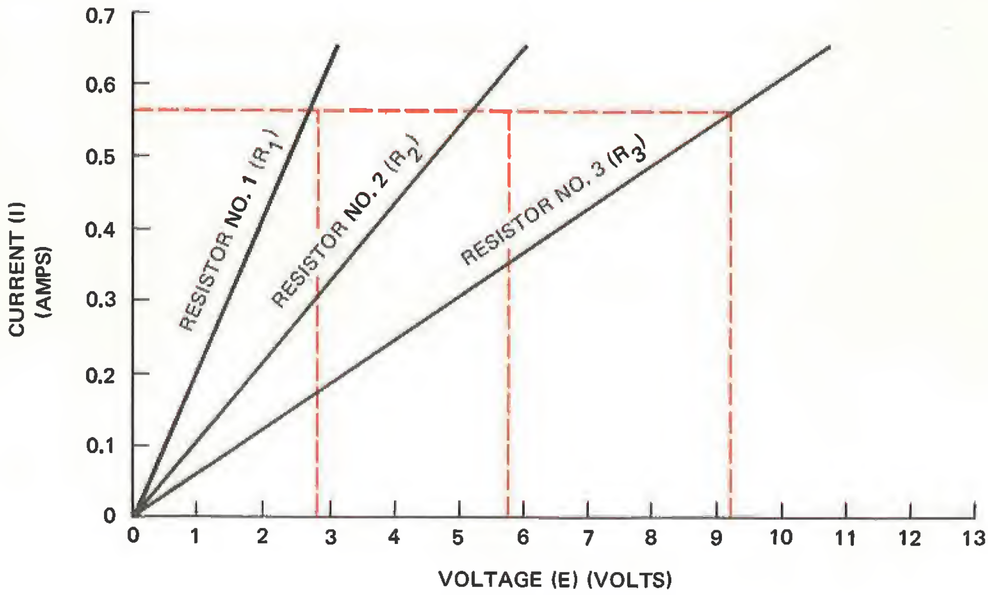


Fig. 3-1 E-I Plots of Linear Resistors

value of current (I - 0.5 amps, for example) and finding the corresponding voltage by projecting the current value onto the E-I plot then down to the voltage axis. (2.5 volts for R_1, 5 volts for R_2, and 10 volts for R_3 in figure 3-1). The resistances may then be computed using Ohm's law (eq 3.1):

$$R_1 = \frac{2.5}{0.5} = 5 \text{ ohms}, \quad R_2 = \frac{5}{0.5} = 10 \text{ ohms},$$

$$R_3 = \frac{10}{0.5} = 20 \text{ ohms}.$$

In the case of a nonlinear resistance, the ohmic value changes as operating conditions change. Or, in other words, the resistance varies as the voltage or current varies. The E-I plot of such a device will not be a straight line. Figure 3-2 shows nonlinear E-I plots.

If we compute the ohmic value of R_2 at several points along the curve, the resistance will be different at each point. Consider, for example, currents of 0.1, 0.3, and 0.6 amps.

$$(\text{for } I = 0.1\text{A}) \quad R'_2 = \frac{5}{0.1} = 50 \text{ ohms}$$

$$(\text{for } I = 0.3\text{A}) \quad R''_2 = \frac{8.5}{0.3} = 28.3 \text{ ohms}$$

$$(\text{for } I = 0.6\text{A}) \quad R_2 = \frac{11}{0.6} = 18.6 \text{ ohms}$$

In addition to changing from point-to-point, the resistance of a nonlinear device may also vary from region-to-region. This regional resistance is called the *dynamic resistance* (r) in the selected region. The dynamic resistance in a region may be determined by dividing the voltage *change* in the range (ΔE) by the resulting current change (ΔI) in the range. That is:

$$r_2 = \frac{\Delta E_2}{\Delta I_2} = \frac{E'_2 - E_2}{I'_2 - I_2} \qquad (3.2)$$

For example, let us compute the dynamic resistance in the range between currents of 0.1 and 0.3 amps.

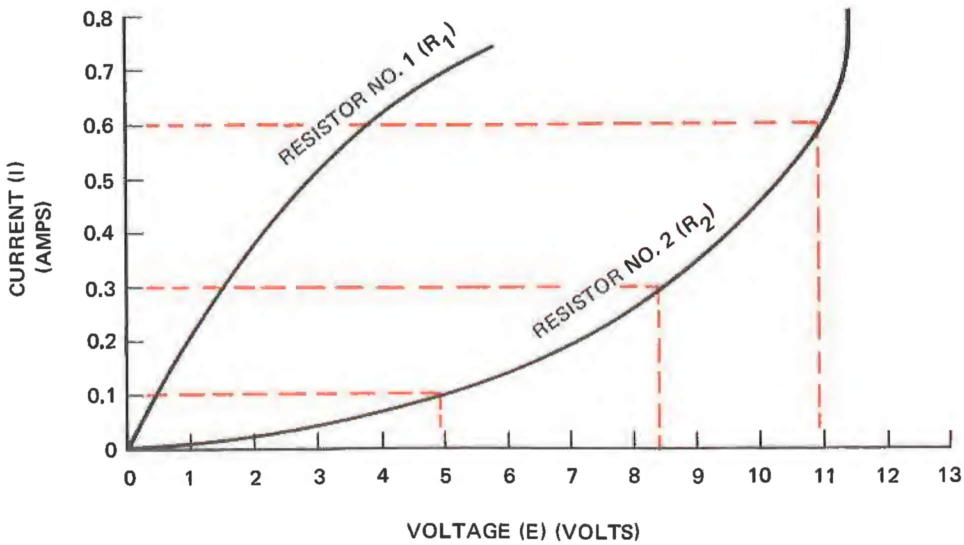


Fig. 3-2 E-I Plots of Nonlinear Resistors

$$r_2 = \frac{\Delta E_2}{\Delta I_2} = \frac{E_2}{I'_2 - I_2} = \frac{8.5 - 5.0}{0.3 - 0.1} = \frac{3.5}{0.2} = 17.5 \text{ ohms}$$

Similarly, the dynamic resistance effective between currents of 0.3 and 0.6 amps is

$$r'_2 = \frac{\Delta E_2}{\Delta I_2} = \frac{E''_2 - E'_2}{I''_2 - I'_2} = \frac{11 - 8.5}{0.6 - 0.3} = \frac{2.5}{0.3}$$

$$= 8.3 \text{ ohms}$$

Notice that the values of dynamic r not only differ from one another, but also differ from the *static resistance* R_2, R'_2, and R''_2. Actually, this method of determining dynamic resistance is only approximate. It will, however, suffice for the present. Investigation of figure 3-1 will reveal that for linear resistors the dynamic and static resistances are equal and constant.

In virtually all cases, it is the increase in temperature resulting from increased current that causes the resistance of a nonlinear device to change. If the resistance increases with an increase in temperature (as R_1 in figure 3-2 does) we say the material has a *positive temperature coefficient.* Conversely, if the resistance decreases with an increase in temperature (R_2 in figure 3-2) we say the material has a *negative temperature* coefficient.

MATERIALS

- 2 Multimeters (VOM)
- 1 DC power supply 0-30V
- 1 150 ohm 10W resistor
- 1 Glo-Bar resistor (GC 25-912 or equiv.)
- 1 28 volt pilot lamp (GE 313 or equiv.)
- 1 Lamp base for pilot lamp
- 3 Sheets of linear graph paper (10 x 10 div. per cm)

PROCEDURE

1. Using one multimeter to measure current and the other to measure voltage, connect the circuit shown in figure 3-3.

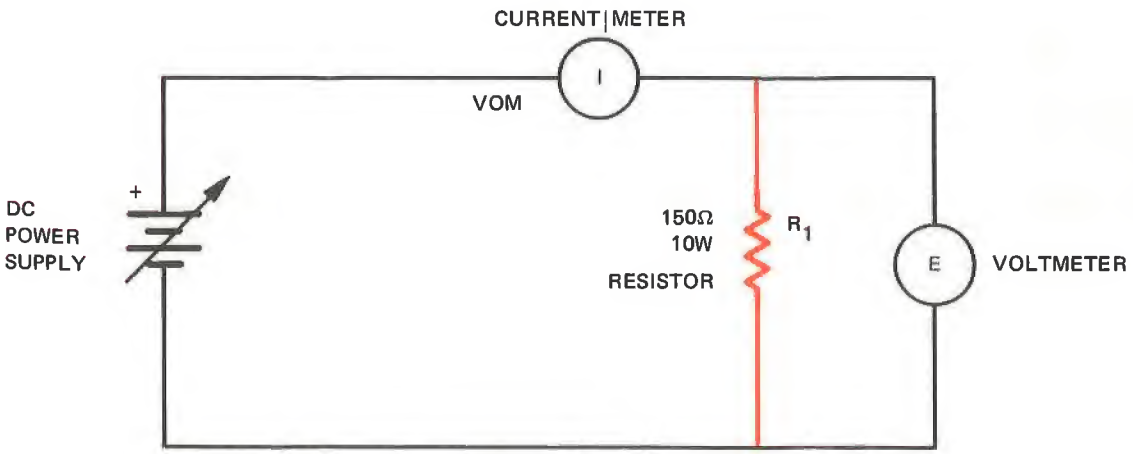


Fig. 3-3 The Experimental Circuit

2. Starting with the DC power supply set for zero output, measure the resistor voltage and current for voltages of 0, 2, 4, 6, ... etc. up to 24 volts. Record this data in the data table (figure 3-4).
   Note: Do Not Exceed A Current Of 0.5 Amps! (glo-bar resistor may!)

3. From the measured values of voltage and current compute the resistance at each data point. Record the values in the data table.

4. Using the measured quantities, compute and record the dynamic resistance in each interval between data points.

150 Ohm Resistor				Glo-Bar Resistor				28V Pilot Lamp			
E	I	R_1	r_1	E	I	R_2	r_2	E	I	R_3	r_3

Fig. 3-4 The Data Table

5. Replace the 150 ohm resistor with the glo-bar resistor and repeat steps 2, 3, and 4.
6. Replace the glo-bar resistor with the 28V pilot lamp and repeat steps 2, 3, and 4.
7. On a single sheet of linear graph paper, plot the volt-ampere (E-I) curve of the three elements.
8. Using a second sheet of graph paper, plot the voltage versus the static resistance of the three elements. Plot voltage as the abscissa and the resistance as the ordinate.
9. On a third sheet of graph paper, plot the voltage versus the dynamic resistance of the three elements.

ANALYSIS GUIDE. The curves plotted in this experiment graphically depict the differences between linear and nonlinear resistances. In your analysis of the data, you should consider both the type of differences that are revealed and the extent of these differences.

PROBLEMS

1. A certain heating element requires 3.2 amps when it is connected to 120 volts. What is the resistance of the element?
2. If the heating element in problem 1 has a cold resistance of 76 ohms, is its temperature coefficient positive or negative?
3. A current meter reads 1.2 amperes when there is 50mV across its terminals. What is the resistance of the meter?
4. When a 1000 ohm resistor is connected to a power supply, 0.33 amps is observed to flow. A second unknown resistor is connected to the same source and 0.1 amps flow. What is the value of the unknown resistor?

experiment 4 ELECTRIC POWER

INTRODUCTION. As an electric current flows through a conductor, some of the energy being conveyed by the current is dissipated in the form of heat. In this experiment, we shall examine this dissipation and in particular we shall consider how it is related to the circuit voltage, current, and resistance.

DISCUSSION. Power can be described as the rate at which energy is converted from one form to another. In the case cited above, the conversion was from electrical to thermal energy. Power may be defined algebraically as

$$P = \frac{W}{t}$$

where P is the electrical power (in watts), W is the energy converted (in joules), and t is the time during which conversion takes place (in seconds).

At this point, we should recall that the current involved in the conversion is defined as

$$I = \frac{Q}{t}$$

from which

$$t = \frac{Q}{I}$$

where I is current, and Q is charge in coulombs. Substituting this expression for t into the power definition gives us:

$$P = \frac{WI}{Q}$$

However, we know that voltage can be defined as:

$$E = \frac{W}{Q}$$

Therefore, power can be determined by

$$P = EI \qquad (4.1)$$

Or in other words, the electrical power dissipated in a circuit is equal to the product of the circuit voltage and the circuit current.

An alternate relationship may be determined by recalling that Ohm's law provides that

$$E = IR$$

which, if substituted into eq. 4.1, provides

$$P = I^2R \qquad (4.2)$$

Similarly from Ohm's law

$$I = \frac{E}{R}$$

Substituting this relationship into eq. 4.1 leads to

$$P = \frac{E^2}{R} \qquad (4.3)$$

These three equations (4.1, 4.2, and 4.3) are frequently referred to as the basic electrical power relationships. They provide us with three convenient alternate ways of determining the power being converted in an electric circuit. To illustrate the use of these three relationships, consider the circuit shown in figure 4-1.

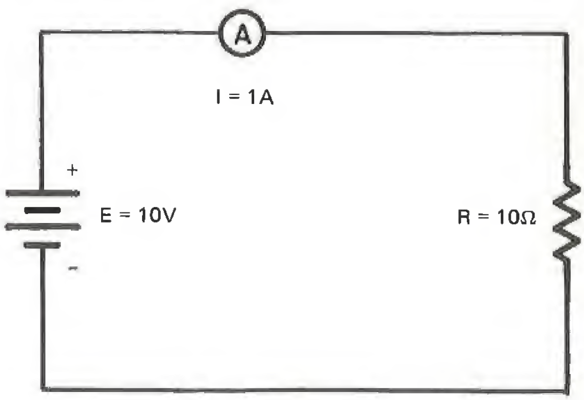


Fig. 4-1 A Simple Electric Circuit

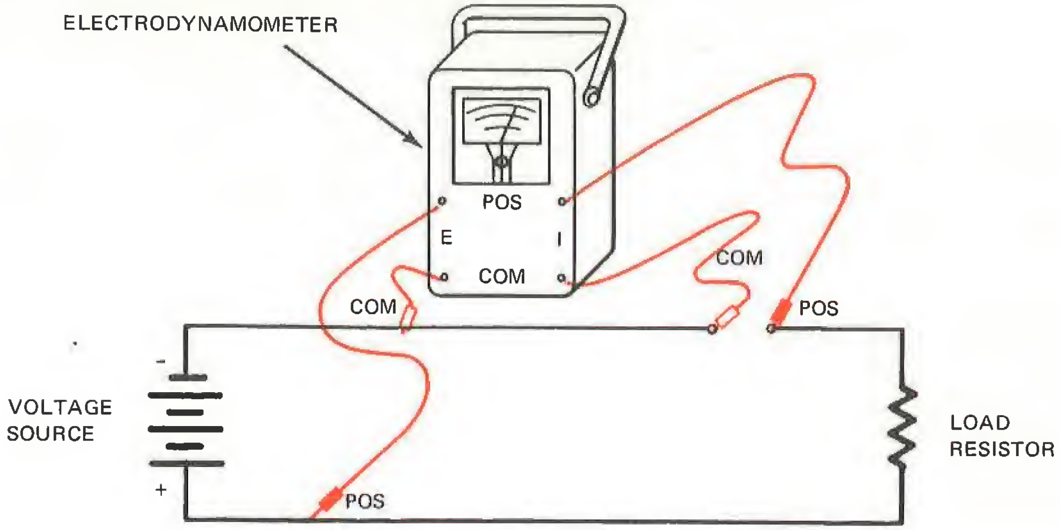


Fig. 4-2 Connecting the Wattmeter

If the loss in the wires is ignored, the power dissipated by the resistor may be determined with any one of the three basic power equations.

$$P = EI = 10 \times 1 = 10 \text{ watts}$$

$$P = I^2R = 1 \times 1 \times 10 = 10 \text{ watts}$$

$$P = \frac{E^2}{R} = \frac{10 \times 10}{10} = 10 \text{ watts}$$

The instrument most commonly used to measure electrical power is the electrodynamometer. In effect, the instrument employs equation 4.1. That is, it measures both voltage and current and indicates their product. Since both a voltage and current measuring connection is required, some caution in connecting the *wattmeter* is necessary. Figure 4-2 shows the proper connection of an electrodynamometer.

As in the case of an ammeter, the current terminals of the wattmeter must be connected so that the circuit current passes through the instrument. The voltage terminals on the other hand must be connected *across* the appropriate part of the circuit. Many wattmeters are constructed in such a way that one terminal of the voltage and current coils is *common.* (They are connected together inside the instrument). When using a meter of this type, the common connections *must* be attached to the *same* point in the test circuit.

In some cases, either the voltage or current connections may be attached to the circuit in a way which causes the meter pointer to deflect backwards from zero. In such an event, either the current or the voltage connection of the wattmeter should be reversed. Reversing *both* the voltage and current connections will result in the same backward deflection.

The current measuring section of the electrodynamometer is very much like any other ammeter in that it has an upper current limit which must not be exceeded. As with other ammeters, it is possible to damage the instrument by allowing too much current to flow through it. In the case of an ammeter, however, the violent off-scale reading serves as a warning to the user that the current is too large. The electrodynamometer, on the other hand, reads out the product of the voltage and current. It is therefore possible to have a very large current and a small voltage, the product of which does not exceed the power range of the instrument. *The wattmeter therefore may not warn the user of an over-current condition.*

MATERIALS

- 1 Multimeter (VOM)
- 1 Electrodynamometer (0-20 watts)
- 1 75 ohm 20 watt resistor
- 1 DC power supply (0-30 volts)
- 1 Vacuum tube voltmeter (VTVM)

PROCEDURE

1. Assemble the test circuit shown in figure 4-3. DO NOT TURN THE POWER SUPPLY ON
2. Have the laboratory instructor check your circuit for correct connection, and initial your data table (fig. 4-4).
3. Turn on the power supply and advance the voltage sufficiently to cause a slight deflection on each instrument. Observe each meter to insure that it is deflecting properly.
4. Increase the voltage until the voltmeter reads about 20 volts. Record the value of the voltage (E) in column 1 of the data table.
5. Measure and record the values of the circuit current and power (I and P_1) in column 1 of the data table.
6. Remove the resistor from the circuit and measure its ohmic value. Record this value as R in column 1 of the data table. Replace the resistor in the circuit.
7. Carefully disconnect the power supply wires and reconnect them with the opposite polarity. Record the reading of the wattmeter as P'_1 in column 1. Observe the effect of reversed polarity on the voltmeter and ammeter then return the connections to the original polarity.

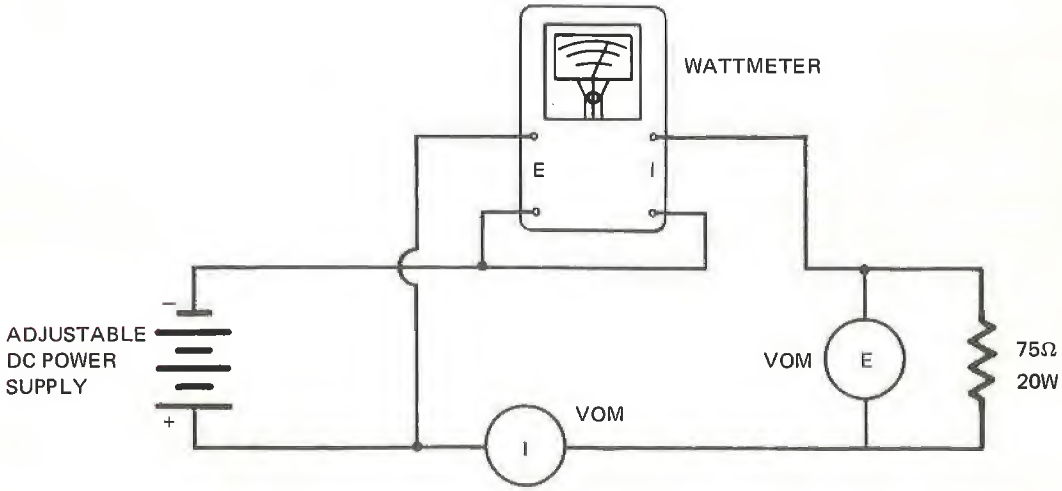


Fig. 4-3. The Experimental Circuit

8. Increase the power supply output until the voltmeter reads about 25 volts. Record the voltage reading as E in the data table column 2.
9. Repeat steps 5 and 6, recording the data as I, R and P_1 in column 2 of the data table.
10. Again increase the power supply output, this time until the voltmeter reads about 30 volts. Record the value of the voltage as E in column 3.
11. Repeat steps 5 and 6, entering these data as I, R and P_1 in the data table column 3.
12. Using the values of E and I in column 1, compute and record $P_2 = EI$.
13. Similarly, using E and R, compute and record in column 1 $P_4 = I^2R$.
14. In like manner, compute and record $P_4 = I^2R$.
15. Repeat steps 12, 13, and 14 first for column 2 values and then for column 3 values.

	Column 1	Column 2	Column 3
E			
I			
R			
P_1			
P_2			
P_3			
P_4			
P_i			

Fig. 4-4 The Data Table

Circuit checked by____________________
(Instructor)

ANALYSIS GUIDE. In drawing your conclusions from the results of this experiment, you should consider whether or not there was good agreement between the measured and computed values of power in each of the three cases. How does your data support your conclusions? In step 6, you observed the effect of reversed polarity on each of three meters employed; how do you explain these effects? Did the resistor value change during the experiment? (Use your data to support your answers.)

PROBLEMS

1. Assume that a resistor of *exactly* 75 ohms has been used in the experiment and that the applied voltage has been exactly 20, 25, and 30 volts. Prepare a new data table and fill in values for each space computed from these assumptions.
2. Compare your idealistic data to those collected in the experiment. How well does each compare?

5 SERIES RESISTANCES

INTRODUCTION. Among the several ways in which electric circuit elements may interconnect is one which requires the *same* current to flow through two or more components. Such an arrangement is called a *series circuit.* In this experiment, we shall examine several of the characteristics of simple series circuits.

DISCUSSION. As mentioned above, a series electrical circuit is one in which the same current must flow through two or more components. While this defining statement is a bit difficult to visualize, it *is* very important. Figure 5-1 should make it somewhat more understandable.

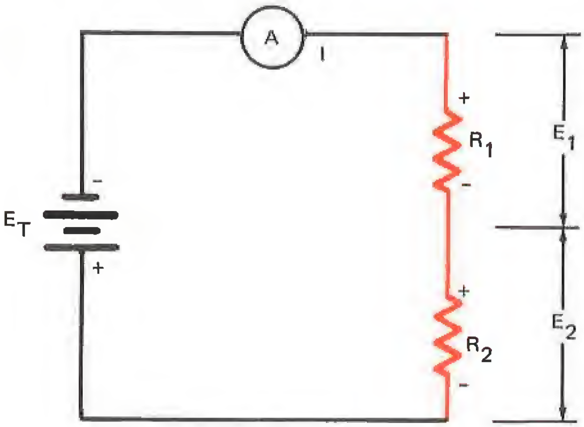


Fig. 5-1 A Simple Series Circuit

Referring to the figure, one observes that all of the current flowing from the source must flow through both R_1 and R_2, then return to the source. This is the type of arrangement which is called a *series* circuit.

Some insight into the characteristics of series circuits may be gained by considering the voltage across each element. Applying Ohm's law to each of the resistors, we see that:

$$E_1 = IR_1 \quad \text{and} \quad E_2 = IR_2$$

However, inspection of the circuit reveals that the voltage across both resistors is the sum of these two voltages. At the same time, the voltage across both resistors must equal the source voltage. Expressed algebraically, this condition is:

$$\boxed{E_T = E_1 + E_2} \qquad \textbf{(5.1)}$$

Or in other words, the sum of the voltage drops in a series circuit must equal the applied voltage.

This condition is frequently expressed in an alternate form as:

$$E_1 + E_2 - E_T = 0$$

This relationship is frequently called *Kirchhöff's voltage law* and may be stated as: *The algebraic sum of the voltages around a series circuit is equal to zero.* It should be observed that this is really nothing more than an alternate way of describing eq. 5.1.

If we return to equation 5.1 and recall that $E_T = IR_T$, $E_1 = IR_1$, and $E_2 = IR_2$ we have

$$IR_T = IR_1 + IR_2$$

or by factoring out the I on the right side

$$IR_T = I(R_1 + R_2)$$

Then if each side of the equation is divided by I, the result is

$$\boxed{R_T = R_1 + R_2} \qquad \textbf{(5.2)}$$

which tells us that *the total resistance in a series circuit is equal to the sum of the series resistors.*

In some cases, we may wish to know the voltage across only one of the resistors in a circuit, say E_2 in figure 5-1 for example. All that is necessary is that we observe that

$$E_2 = IR_2$$

and

$$I = \frac{E_T}{R_T}$$

Combining these two relations gives us:

$$E_2 = \frac{E_T}{R_T} R_2$$

However, since $R_T = R_1 + R_2$ (eq. 5.2), we may write E_2 in the form:

$$E_2 = E_T \frac{R_2}{R_1 + R_2} \quad (5.3)$$

This equation describes what is frequently called the *voltage divider* action of the series circuit and the fraction $R_2/(R_1 + R_2)$ is called the *voltage division ratio.*

By applying very similar steps, we can also arrive at the relationship for E_1. The result in the case of E_1 would be:

$$E_1 = E_T \frac{R_1}{R_1 + R_2} \quad (5.4)$$

Electrical power is supplied to the circuit from the source and is distributed among the circuit resistances. The way in which the power is distributed may be investigated by returning to equation 5.1:

$$E_T = E_1 + E_2$$

If we multiply each term in the equation by I the result is:

$$E_T I = E_1 I + E_2 I$$

Since $E_T I$ is equal to the total power (P_T) supplied by the source while $E_1 I$ is the power (P_1) dissipated by R_1 and $E_2 I$ is the power (P_2) dissipated by R_2, we may write

$$P_T = P_1 + P_2 \quad (5.5)$$

for the simple series circuit of figure 5-1.

Series circuits are at times constructed using both linear and nonlinear elements. Such nonlinear circuits are very frequently found in electronic applications. Figure 5-2 shows a very simple example of a circuit of the nonlinear type. These circuits may be analyzed in a number of ways, but the most popular approach is a graphical one.

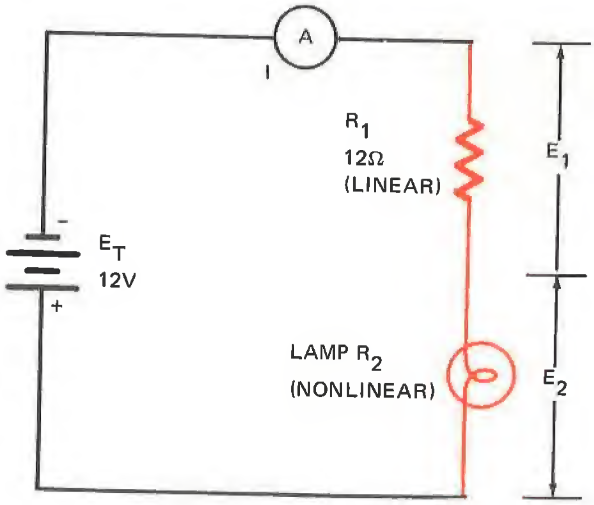


Fig. 5-2 A Nonlinear Series Circuit

Let us suppose that figure 5-3 is the E-I plot of the nonlinear element (R_2) in the circuit. (Ignore for the moment the straight construction lines.)

The curved line represents all of the values of I which can flow through R_2 for values of E_2 within the range of the curve. If we could plot another line representing the values of I through R_1 *for various values of E_2*, we could find the single value of I which would satisfy the requirements of both R_1 and R_2. This second line is called the *load line* of R_1 and may be identified by rearranging equation 5.1 to the form:

$$E_1 = E_T - E_2$$

And since $E_1 = IR_1$ while $E_2 = IR_2$, we have:

$$IR_1 = E_T - IR_2$$

By dividing each term by R_1, the relationship becomes:

$$I = \frac{E_T}{R_1} - I\frac{R_2}{R_1}$$

However, we know that $I = E_2/R_2$. Substituting this fraction for I on the right side and simplifying, renders

$$I = \frac{E_T}{R_1} - \frac{1}{R_1} E_2$$

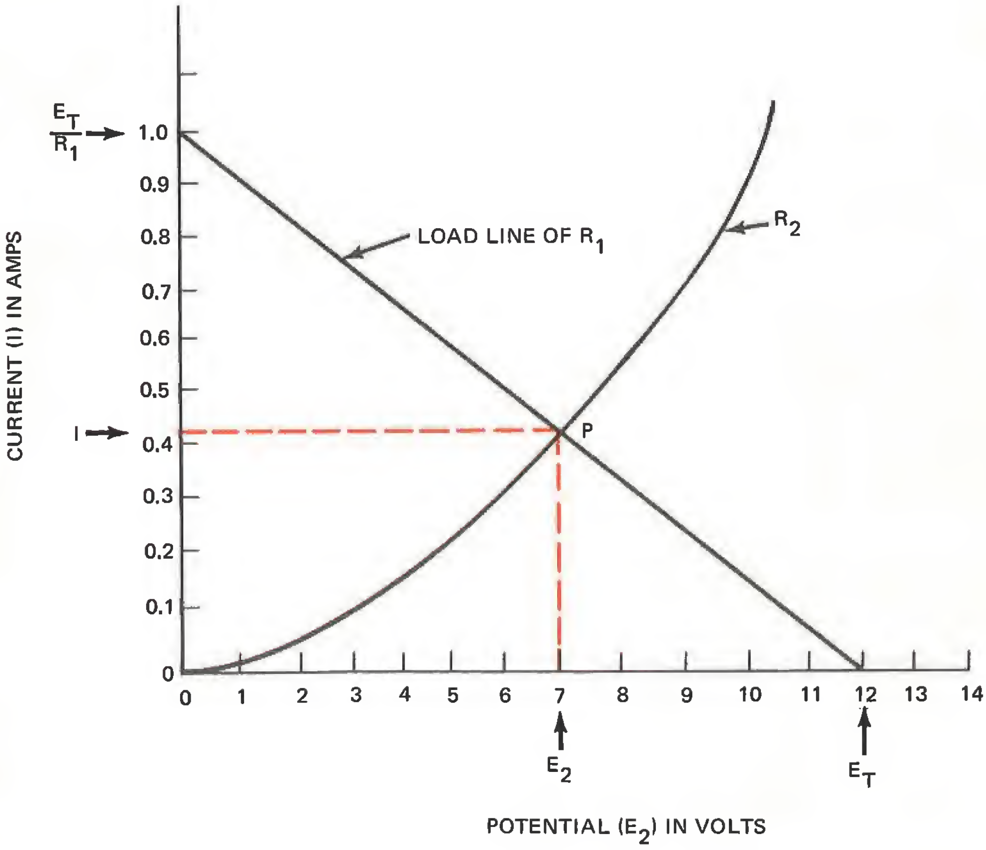


Fig. 5-3 Loadline Analysis of Nonlinear Circuit

This may be rearranged into the form

$$I = -\frac{1}{R_1} E_2 + \frac{E_T}{R_1}$$

This is an equation of the form y = mx + b, which is a straight line. The slope of the line (m) is – $1/R_1$ the y intercept (b) is E_T/R_1 while the x intercept (found by letting I equal zero and solving for E_2) is at $E_2 = E_T$. *This is the load line of R_1 and E_T. It may be plotted by drawing a straight line from the value of E_T on the voltage axis to the value E_T/R_1 on the current axis of the R_2 E-I plot.* (See figure 5-3)

With the load line of R_1 constructed, the values of the circuit current (I) and the potential across R_2 (E_2) may be read by projecting the intersection of the R_2 E-I plot and the load line onto the two axes, as shown in figure 5-3. The value of E_1 may then be determined using equation 5.1 or Ohm's law.

MATERIALS

- 2 Multimeters (VOM)
- 1 Wattmeter (0-20W)
- 1 Variable DC power supply
- 1 150 ohm resistor
- 1 75 ohm resistor
- 1 28V pilot lamp
- 1 Lamp base
- 1 Sheet of linear graph paper

PROCEDURE

1. Assemble the simple series circuit shown in figure 5-4.

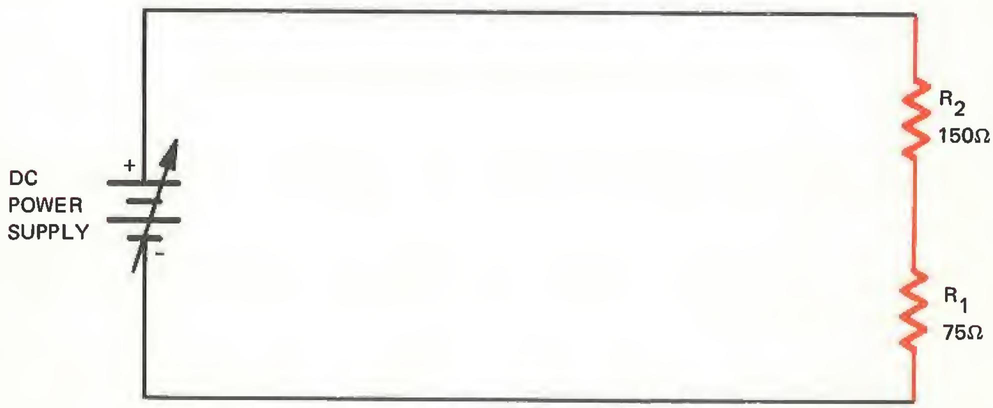


Fig. 5-4 The Linear Experimental Circuit

2. Set the power supply output to about 30 volts. Measure and record this quantity as E_T in the data table. (Figure 5-6)

3. Measure and record the circuit current (I) in the data table.

4. Measure the voltage across each of the resistors and record the value across the 150 ohm resistor as E_2 and the value across the 75 ohm resistor as E_1.

5. Using the wattmeter measure and record the total input power (P_T), the power dissipated by the 75 ohm resistor (P_1), and the power dissipated by the 150 ohm resistor (P_2).

6. Disassemble the circuit and measure the ohmic value of each resistor. Record these values as R_1 and R_2 for the 75 and 150 ohm resistor respectivelv. Also measure and record the value of R_T, the total series resistance.

7. Set up the circuit shown in figure 5-5.

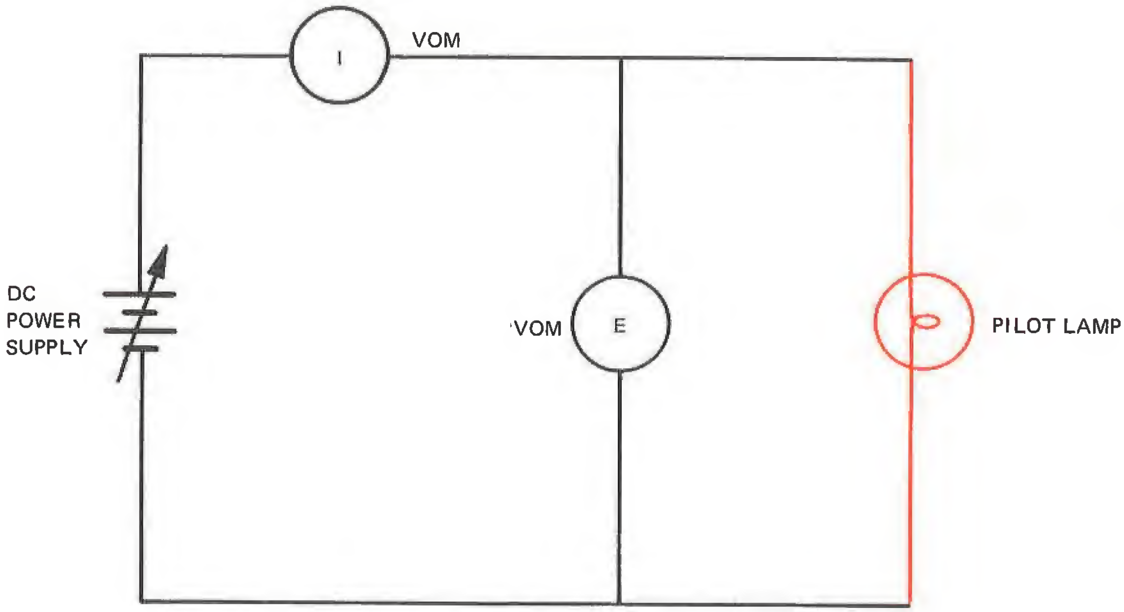


Fig. 5-5 The Nonlinear Experimental Circuit

8. Starting at zero, measure and record the lamp voltage and current every 3 volts up to 30 volts.

9. Remove the meters from the circuit and connect the 75 ohm linear resistor in series with the bulb across the source.

10. Set the source voltage to about 30 volts. Measure and record this value as E'_T in the data table.

11. Measure and record the circuit current (I'), the resistor voltage (E'_1), and the bulb voltage (E'_2).

12. Using the specified values of R_1 and R_2, compute and record the value of R_T.

13. From the computed value of R_T and the measured value of E_T, compute and record the value of the circuit current I.

14. Use equation 5.4 to compute the value of E_1. In this calculation, use E_T and the computed value of R_T.

15. From the value of E_T and the computed E_1, determine the value of E_2 using equation 5.1.
16. Using only computed values, calculate P_1 and P_2.
17. With the results of step 16 and equation 5.5, compute the value of P_T.
18. On a sheet of linear graph paper, plot the E-1 curve of the lamp.
19. Construct the 75 ohm load line on the E-I curve and determine I′ and E'_2. Enter these data in the "computed" column of the data table.
20. With the results of step 19, compute the value of E'_1.

LINEAR CIRCUIT VALUES		
Qty	Measured	Computed
E_T		X
I		
E_1		
E_2		
P_T		
P_1		
P_2		
R_1		X
R_2		X
R_T		

NONLINEAR CIRCUIT VALUES		
Qty	Measured	Computed
E'_T		X
I′		
E'_1		
E'_2		

E-I PLOT DATA

E Volts	0	3	6	9	12	15	18	21	24	27	30
I											

Fig. 5-6 The Data Table

ANALYSIS GUIDE. In considering the results obtained in this experiment, you should compare the measured data to the computed data in each case. Are the two values relatively close to each other? Do your results indicate that the techniques used in computation were reasonably effective?

PROBLEMS

1. Three resistors of 100, 150, and 200 ohms are connected in series across a voltage source. If the power dissipated by the 150 ohm resistor is 0.5 watts, what is the voltage across the 200 ohm resistor?
2. What is the value of the source voltage in problem 1?
3. Derive the equation for E_1 given in the discussion as eq. 5.4.
4. What is the power output of the source in problem 1?

experiment 6 SOURCE RESISTANCE

INTRODUCTION. It would be convenient if every electrical source were perfect and did not affect the operation of circuits attached to it. Unfortunately, this is not the case. In this experiment we shall examine one of the characteristics of a source which does affect external circuit operation.

DISCUSSION. When the current drawn from an electrical source varies over a relatively wide range, the terminal voltage of the source tends to vary also. This variation in source voltage with current can be explained by considering a resistance to be in series with the source voltage. Figure 6-1 shows a schematic representation of this situation.

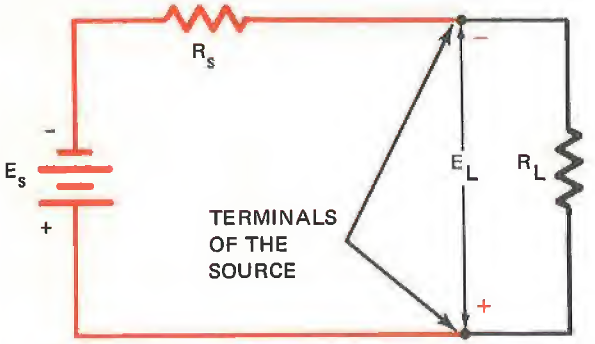


Fig. 6-1 A Source With Internal Resistance

By applying Ohm's law, we see that the voltage developed across the internal source resistance is equal to IR_s. Kirchhoff's law can be applied to the loop to determine that the load voltage is

$$E_L = E_s - IR_s$$

Solving for R_s renders

$$R_s = \frac{E_s - E_L}{I} = \frac{E_s}{I} - \frac{E_L}{I}$$

But since $I = E_L/R_L$, we can write

$$R_s = \frac{E_s R_L}{E_L} - \frac{E_L R_L}{E_L}$$

If we cancel E_L on the extreme right and factor R_L out of both righthand terms, the result is

$$R_s = R_L\left(\frac{E_s}{E_L} - 1\right) \qquad \textbf{(6.1)}$$

From this relationship, we see that the value of R_s may be determined if the load resistance, source voltage, and load voltage are known. The source voltage is the only one of these quantities which presents a measurement problem. Since we normally cannot go inside a source, the value of E_s must be measured at the source terminals. This can only be done when no current is flowing from the source. However, in most practical cases, if the terminal voltage is measured with *no load,* the value will be sufficiently close to the actual value of E_s that it can be used for determining R_s.

The presence of the internal source resistance in series with any practical voltage supply has a definite effect on the amount of power that a source can deliver to a load. The amount of power dissipated by the load in figure 6-1 will be

$$P_L = \frac{E_L^2}{R_L} \qquad \textbf{(6.2)}$$

However, voltage divider action requires E_L to be

$$E_L = \frac{R_L}{(R_s + R_L)} E_s$$

Squaring this expression and substituting it for E_L^2 in the power expression gives:

$$P_L = E_s^2 \frac{R_L}{(R_s + R_L)^2} \qquad (6.3)$$

If E_s and R_s are constant nonzero values, we see that the load power is not simply inversely proportional to R_L as it would be if R_s were zero. The nature of the relationship between R_L and P_L can perhaps best be understood by plotting the power as a function of the resistance. Figure 6-2 shows such a plot (the solid line). We observe from the plot that as the resistance increases from zero, the power initially increases reaching a maximum when $R_L = R_s$, then decreases as R_L increases.

Returning to the circuit diagram (fig. 6-1) it should be observed that the total power is the sum of the load power and the power dissipated by the internal resistance. That is

$$P_T = P_L + P_s$$

Or in other words, the total power will in general *not* equal the load power. The difference between the two is the power dissipated as heat within the source. It is this heat which can cause damage to a source under overload conditions.

The value of the total power can be determined by taking the ratio of the source voltage squared (E_s^2) to the total circuit resistance. Expressed algebraically this is

$$P_T = \frac{E_s^2}{(R_s + R_L)} \qquad (6.4)$$

Since the total power and the load power differ, the question of efficiency arises. If we define percent of efficiency as

$$\%\ \text{eff.} = \frac{P_L}{P_T} \times 100 \qquad (6.5)$$

and substitute the relationships given in equations 6.3 and 6.4 for P_L and P_T respectively, the percent of efficiency becomes:

$$\%\ \text{eff.} = \frac{R_L}{R_s + R_L} \times 100 \qquad (6.6)$$

A plot of the percent of efficiency is shown in figure 6-2 (dotted line). It should be observed that at maximum power the efficiency is at 50%.

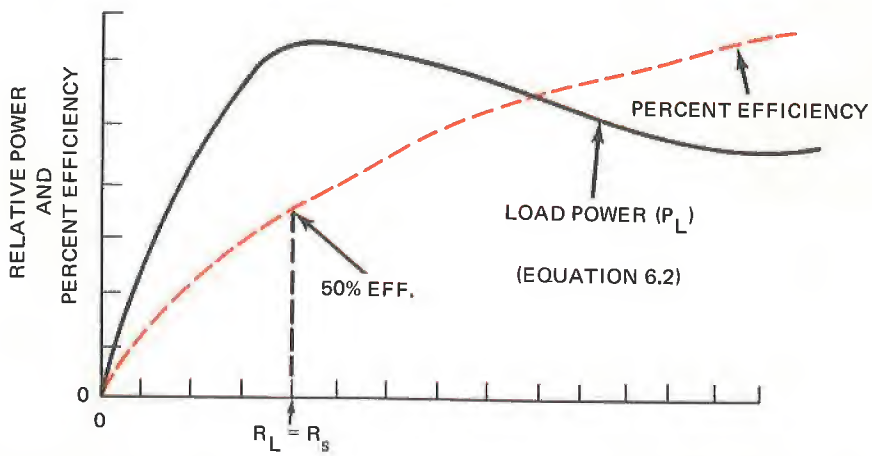


Fig. 6-2 Load Resistance (R_L)

MATERIALS

1 Multimeter

1 Audio signal generator

1 Resistance decade box

2 Sheets of linear graph paper

PROCEDURE

1. Inspect the audio signal generator and determine how to turn it off and on, adjust the output level, and set the frequency of the output.

2. Set up the multimeter for measuring AC voltage.

3. Turn on the audio signal generator and set the frequency for 100 Hz.

4. Connect the multimeter across the audio signal generator output and adjust the level to about 10 volts. Record this value in the data table (Fig. 6-2) as E_S.

5. Set the resistance decade box for 2000 ohms of resistance.

6. Connect the resistance decade box across the audio signal generator output and leave the multimeter connected as before. Record the output voltage (E_L) in the data table opposite the R_L = 2000 ohms entry.

7. Set the resistance decade box to 1800 ohms and record the output voltage (E_L).

8. Repeat step 7 for resistance decade settings of 1600, 1400, 1200, 1100, 1000, 900, 800, 700, 600, 500, 400, 300, 200, 100, 50, and 0 ohms.

9. Immediately disconnect the resistance decade box.

10. Compute the value of the internal resistance of the audio signal generator for each data point and enter the results in the appropriate data table space. Use eq. 6.1 for this calculation.

11. Compute the value of the load power for each data point and record the values in the data table. (Use eq. 6.2)

12. Using equation 6.4, compute and record the total power being produced by the audio signal generator at each data point.

13. On a sheet of linear graph paper, plot a curve of the load resistance versus the total power. Plot resistance on the x axis and power on the y axis.

		E_s = Volts			
R_L	E_L	R_s	P_L	P_T	% eff.
2kΩ					
1.8kΩ					
1.6kΩ					
1.4kΩ					
1.2kΩ					
1.1kΩ					
1.0kΩ					
900Ω					
800Ω					
700Ω					
600Ω					
500Ω					
400Ω					
300Ω					
200Ω					
100Ω					
50Ω					
0					

Fig. 6-2 The Data Table

14. On the same sheet of graph paper, plot a curve of load resistance versus load power.
15. Compute the percent of efficiency for each data point using equation 6.6 and record the values in the data table.
16. Using a second sheet of linear graph paper, plot the load resistance versus efficiency.

ANALYSIS GUIDE. In your analysis you should consider to what extent your results were consistent with the theoretical ones. If your results differ from the theoretical, explain why they did.

PROBLEMS

1. What percentage of the maximum total power did the load power reach? Why is this value different from the efficiency at maximum load power?

2. If one horsepower is equivalent to 746 watts, what would be the value of current drawn by a 117 volt electric motor if the motor output was 1/4 horsepower and its efficiency was 90%?

3. A certain 90-volt battery supplies 84 volts to a 1000 ohm load. What would be the voltage across a 500 ohm load?

4. What would be the maximum power that the battery in problem 3 could supply to a load?

INTRODUCTION. It is possible to interconnect electric circuit elements in such a way that the same voltage *must* appear across two or more elements. In this experiment we shall examine the characteristics of this type of circuit.

DISCUSSION. If the resistance of the interconnecting wires is negligible, the circuit shown in figure 7-1 is arranged so that the same voltage is applied across each of the resistors.

This type of circuit connection is called a *parallel circuit.* Examination of the circuit reveals that the three *branch currents* must all come from the source. The sum of these currents will therefore equal the total source current. Or, in algebraic terms:

$$I_T = I_1 + I_2 + I_3 \qquad (7.1)$$

Ohm's law requires that the various currents be related to the source voltage and the resistances by

$$I_1 = \frac{E_T}{R_1} \qquad I_2 = \frac{E_T}{R_2}$$

$$I_3 = \frac{E_T}{R_3} \qquad I_T = \frac{E_T}{R_T}$$

where R_T is the total effective resistance of the parallel resistors. Substituting these relationships into eq. 7.1 gives us:

$$\frac{E_T}{R_T} = \frac{E_T}{R_1} + \frac{E_T}{R_2} + \frac{E_T}{R_3}$$

But since all of the E_T s are equal, we may divide each term by E_T and have:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (7.2a)$$

If we take the reciprocal of each side of the equation, we arrive at:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \qquad (7.2b)$$

Either equation, 7.2a or 7.2b, allows us to determine the value of the total effective resistance of the circuit.

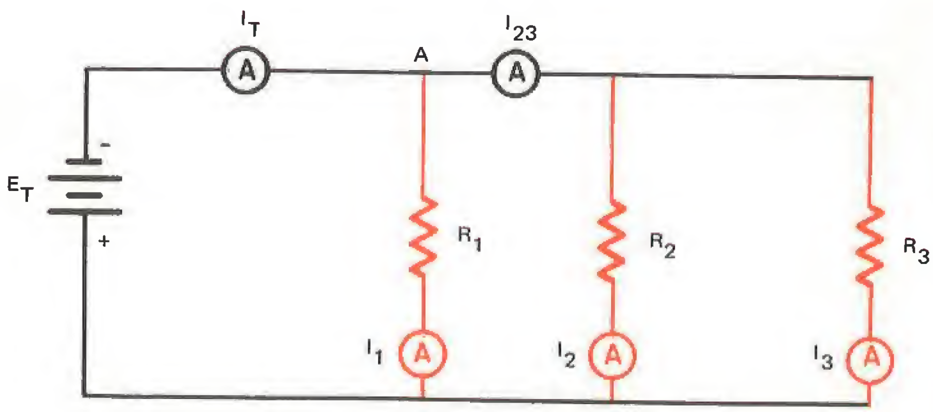


Fig. 7-1 A Parallel Circuit

A special case which is of particular interest arises when only two resistors are connected in parallel. In such a case, equation 7.2a becomes

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Taking the lowest common denominator on the right gives us

$$\frac{1}{R_T} = \frac{R_2 + R_1}{R_1 R_2}$$

Then taking the reciprocals, we have

$$\boxed{R_T = \frac{R_1 R_2}{R_1 + R_2}} \qquad (7.3)$$

This form is perhaps more convenient to use than is eq. 7.2a or 7.2b. In a circuit containing more than two resistors, eq. 7.3 can be used by successively considering pairs of resistors. For example, in figure 7-1, R_2 and R_3 can be combined and then their result combined with R_1, thereby arriving at the total effective resistance for the whole circuit.

The power in a parallel circuit will distribute itself among the various elements. To examine the way in which the power is distributed, let us return to eq. 7.1:

$$I_T = I_1 + I_2 + I_3$$

If we multiply every term by the source voltage, E_T, the result will be:

$$E_T I_T = E_T I_1 + E_T I_2 + E_T I_3$$

However, $E_T I_T$ is the power supplied by the source while $E_T I_1$, $E_T I_2$, and $E_T I_3$ are power dissipations of R_1, R_2, and R_3 respectively.

We may, therefore, conclude that

$$\boxed{P_T = P_1 + P_2 + P_3} \qquad (7.4)$$

for a parallel circuit.

Let us now consider the current flow at one of the branch junctions of the circuit in figure 7-1. At the top of R_1 is a junction marked with an A. The current (I_{23}) flowing in the line to the right of point A is flowing away from the junction and is equal to ($I_2 + I_3$). I_1 is also flowing away from the junction, while I_T is flowing toward the junction. The law governing current flow at a junction was first formulated by Gustav Kirchhoff and hence is called *Kirchhoff's Current Law.* The law may be stated as follows:

> *If current flow toward a junction is considered to be positive and current flow away from a junction is considered to be negative, then the algebraic sum of the currents at the junction will be equal to zero.*

Applying Kirchhoff's current law to junction A above renders

$$+I_T - I_1 - (I_2 + I_3) = 0$$

or

$$I_T - I_1 - I_2 - I_3 = 0$$

Comparison reveals that this expression of Kirchhoff's current law is identical to equation 7.1.

The current in a parallel circuit divides in a manner similar to voltage division in a series circuit. Let us consider the two-element circuit shown in figure 7-2.

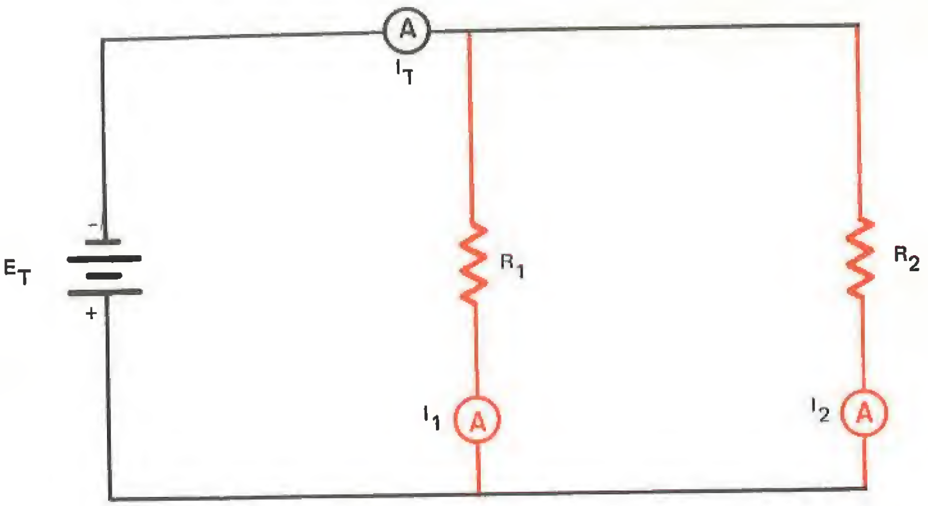


Fig. 7-2 A Two-Element Parallel Circuit

Since the voltage across each element is equal to the applied voltage we see that

$$I_T R_T = I_1 R_1$$

or

$$I_1 = I_T \frac{R_T}{R_1}$$

Substituting the relationship for R_T given in eq. 7.3, we have:

$$I_1 = I_T \frac{R_2}{R_1 + R_2} \qquad (7.5)$$

Alternately, the similar argument for I_2 renders:

$$I_2 = I_T \frac{R_1}{R_1 + R_2} \qquad (7.6)$$

In dealing with parallel circuits, it is sometimes convenient to use the *conductance* of the elements rather than their resistance.

The conductance (G) of an element (in mhos) is given by:

$$G = \frac{1}{R}$$

Equation 7.2a can, therefore, be rewritten using conductances as

$$G_T = G_1 + G_2 + G_3$$

and the total resistance may be found by

$$R_T = \frac{1}{G_1 + G_2 + G_3}$$

Similarly, Ohm's Law may be expressed in terms of conductance as

$$I = EG,\ E = \frac{I}{G},\ \text{or}\ G = \frac{I}{E}$$

Parallel circuits are, on occasion, formed by combining linear and nonlinear elements. A simple circuit of this type is shown in figure 7-3.

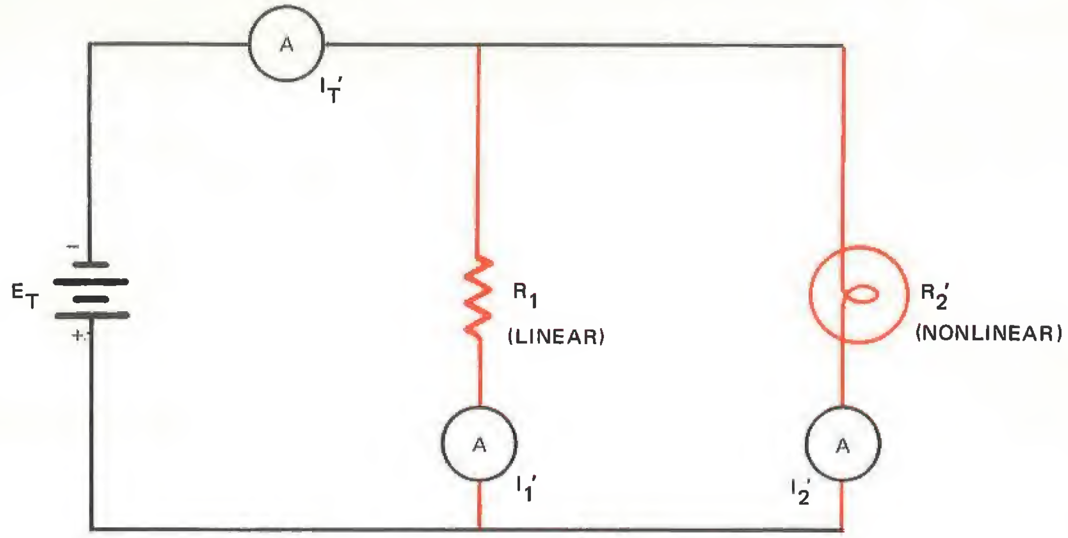


Fig. 7-3 A Nonlinear Parallel Circuit

Analysis of nonlinear parallel circuits is somewhat more involved than simple linear ones. However, some insight into the operation of the circuit may be gained by considering the E-I plot of the two elements. Figure 7-4 shows the E-I plots of a linear and a nonlinear element.

The current flow through R_1 may be determined either by Ohm's Law, $I_1 = E_T/R_1$ or from the E-I plot. The current flow through the nonlinear element may be found from the E-I plot as indicated by the dotted construction on figure 7-4. With the branch currents known, the total current may be determined using Kirchhoff's Current Law. Total effective resistance and power may be found using conventional methods after the currents are known.

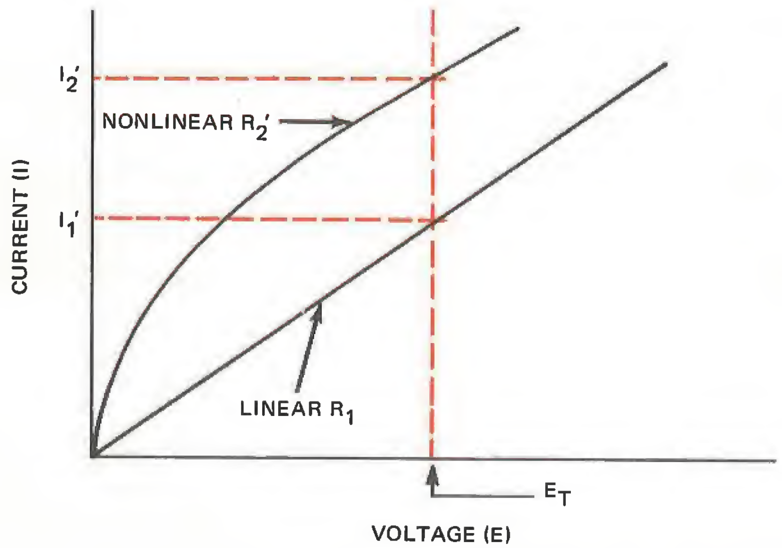


Fig. 7-4 E-I Plots of the Parallel Elements

MATERIALS

1 Variable DC power supply
2 Multimeters (VOM)
1 75 ohm resistor
1 150 ohm resistor
1 250 ohm resistor
1 Glo-bar resistor
1 Sheet of linear graph paper

PROCEDURE

1. Measure the exact values of R_1 (the 250 ohm resistor), R_2 (the 75 ohm resistor), and R_3 (the 150 ohm resistor). Record these values in the data table, figure 7-5.
2. Construct the circuit shown in figure 7-1.
3. Adjust the DC supply for an output voltage of about 15 volts. Record the value (E_T) in the data table.
4. Measure and record the values of I_T, I_1, I_2, I_3, and the current in the line between R_1 and R_2 (I_{23}).
5. Disconnect the source and measure the value of R_T. Record the results in the data table.
6. Remove R_3 (the 150 ohm resistor) and measure the resistance (R_{12}) of R_1 and R_2 in parallel.
7. Construct the circuit shown in figure 7-3 using the 250 ohm resistor (R_1) and the glo-bar resistor (R'_2).
8. Adjust the source voltage to the value established in step 3.
9. Measure and record the currents I'_T, I'_1, and I'_2 in the data table.
10. Disassemble the test circuit and set up the equipment to take E-I plot data for the glo-bar resistor. Construct your own data table to record the values.
11. Measure and record E-I plot data for the glo-bar resistor up to a voltage of 16 volts.
12. Plot the E-I curve of the glo-bar resistor on a sheet of linear graph paper. Using the measured value of R_1, draw (no additional measurements are necessary) the E-I plot on the same set of axes.
13. Using the method described in the discussion, graphically determine I'_1 and I'_2. Using Kirchhoff's Current Law, determine I'_T. Record these data as computed values.
14. Compute the total resistance using eq. 7.2b and record it in the data table (R_T).
15. With eq. 7.3, compute and record the effective resistance (R_{12}) of R_1 and R_2 in parallel.
16. Use the measured value of E_T and the computed value of R_T to compute I_T. Record the result in the data table.

Qty	Measured	Computed
R_1		250 ohms
R_2		75 ohms
R_3		150 ohms
R_{12}		
R_T		
E_T		X
I_1		
I_2		
I_3		
I_{23}		
I_T		
I'_1		
I'_2		
I'_T		

Fig. 7-5 The Data Table

17. Compute I_1 using measured values of E_T and R_1. Record the value in the data table.
18. Apply Kirchhoff's Current Law and the computed values of I_T and I_1 to determine the value of I_{23}. Record the result in the data table.
19. Use the current division relationships and I_{23} to determine I_2 and I_3. Record the values in the data table.

ANALYSIS GUIDE. In this analysis of these data, consider the extent to which the measured values agree with the computed ones. What would be the most likely causes of error in the measured data and how large would these be?

PROBLEMS

1. The total effective resistance of three parallel elements is 27 ohms. One of the elements has a resistance of 93 ohms. The other two have equal resistances. What are their ohmic values?
2. Compute the power dissipation of each resistor in the linear portion of the experiment. Also compute the total power using E_T and I_T. How well do these values conform to the requirements of equation 7.4?

INTRODUCTION. Many practical electric circuits employ both series elements and parallel elements in the same network. In this experiment we shall examine methods of analysis which may be used in this type of circuit.

DISCUSSION. There are a number of alternate approaches one can take to analyze the series-parallel circuit shown in figure 8-1.

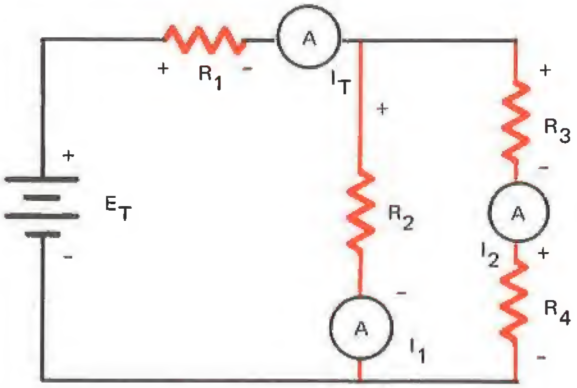


Fig. 8-1 A Series-Parallel Circuit

Perhaps the most traditional method is to start with the elements farthest from the source and simplify the circuit by combining elements. Applying this method to the circuit in figure 8-1, we would first combine the two series elements R_3 and R_4.

$$R_{34} = R_3 + R_4$$

The circuit would then be reduced to the one shown in figure 8-2(a).

The result, R_{34}, is then combined with the parallel element R_2, as shown in figure 8-2(b).

$$R_{234} = \frac{R_2 R_{34}}{R_2 + R_{34}}$$

Finally, the total effective resistance, R_T, is determined by combining R_{234} with the series element R_1.

$$R_T = R_1 + R_{234}$$

Thus reducing the circuit to the single element of figure 8-2(c).

With the total resistance and voltage known, the total current, I_T, can be found using Ohm's Law. The branch currents may then be determined and the voltage drop across any individual element computed.

An alternate method of circuit solution would be to write a circuit equation for the quantity desired. For example, the equation for R_T can be written by starting at the source and reasoning as follows: R_T must equal R_1 plus the parallel combination of R_2 and R_{34}.

$$R_T = R_1 + \frac{R_2 R_{34}}{R_2 + R_{34}}$$

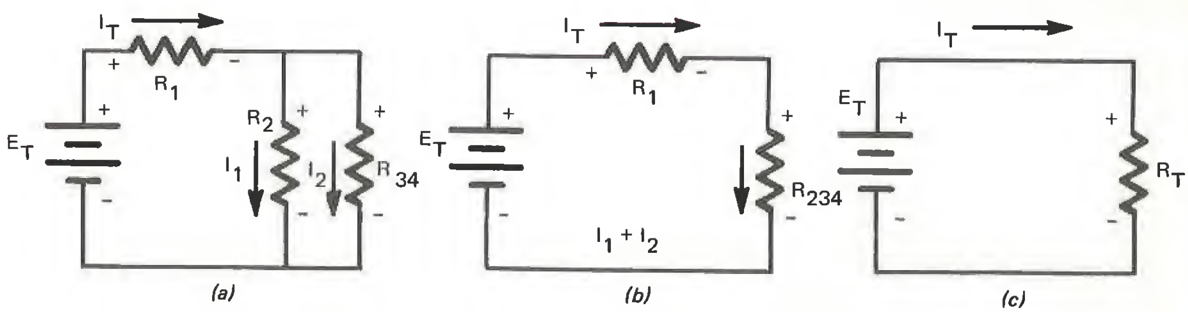


Fig. 8-2 Simplifications of a Series-Parallel Circuit

However, R_{34} is equal to the sum of R_3 and R_4 so R_T will be

$$R_T = R_1 + \frac{R_2(R_3 + R_4)}{R_2 + R_3 + R_4}$$

In like manner, the equation for any other circuit quantity may be derived. Or, in some cases, it is expedient to apply the more traditional approach from this point on.

Analysis of a series-parallel circuit may also be carried out by applying Kirchhoff's voltage and current laws. Since R_2 and R_{34} are in parallel, the voltage (E_2) across R_2 must equal the voltage ($E_3 + E_4$) across R_{34}.

$$E_2 = E_3 + E_4 \qquad (8.1)$$

Similarly, the total voltage (E_T) must equal the sum of the voltages across R_1 and E_2.

$$E_T = E_1 + E_2 \qquad \textbf{(8.2)}$$

Since $E_1 = I_T R_1$, $E_2 = I_1 R_2$, $E_3 = I_2 R_3$, and $E_4 = I_2 R_4$, we can rewrite equations 8.1 and 8.2 as:

$$I_1 R_2 = I_2 R_3 + I_2 R_4 \qquad \textbf{(8.1a)}$$

$$E_T = I_T R_1 + I_1 R_2 \qquad \textbf{(8.2a)}$$

Also, we see at the junction of the branches that

$$I_T = I_1 + I_2 \qquad \textbf{(8.3)}$$

These three equations (8.1a, 8.2a, and 8.3) form a set of three simultaneous linear equations with three unknowns (I_T, I_1, and I_2). They can be solved for the unknown currents using conventional algebraic methods. With the currents determined, the other circuit quantities may be computed.

In both simple series and parallel circuits, the total power dissipated is equal to the sum of power dissipations of the individual elements. This relationship also applies in the case of a series-parallel circuit. That is:

$$P_T = P_1 + P_2 + P_3 + P_4 \qquad \textbf{(8.4)}$$

A frequently encountered application of series-parallel circuits is the one which arises out of the need to supply several voltages from a single source. Figure 8-3 illustrates such an application.

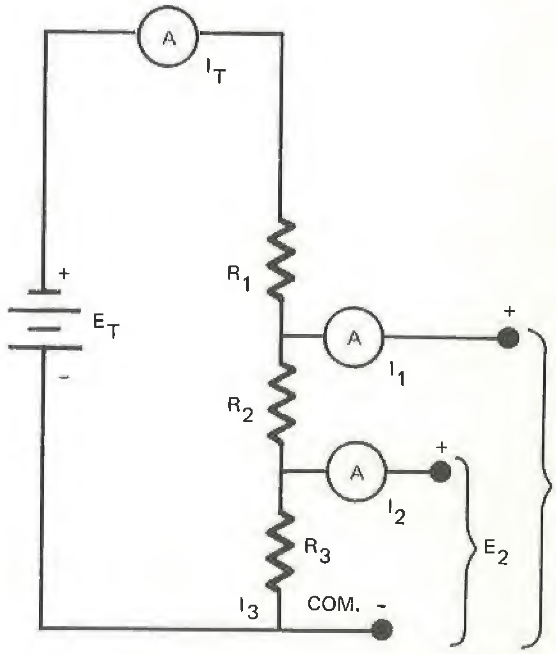


Fig. 8-3 A Voltage Divider Circuit

The analysis of this type of voltage divider circuit is carried out in the same way as any other series-parallel arrangement. However, one should notice that the resistances across which E_1 and E_2 are applied are not shown in the circuit. The value of the loads may of course be determined using

$$R_{L1} = \frac{E_1}{I_1} \quad \text{and} \quad R_{L2} = \frac{E_2}{I_2}$$

MATERIALS

1 Variable DC power supply
1 Multimeter
1 75 ohm resistor
1 100 ohm resistor
1 150 ohm resistor
1 250 ohm resistor

PROCEDURE

1. Measure and record in the data table, figure 8-5, the value of each of the resistors.
2. Assemble the circuit shown in figure 8-4 and measure the total effective resistance of the network. Record the value in the data table.

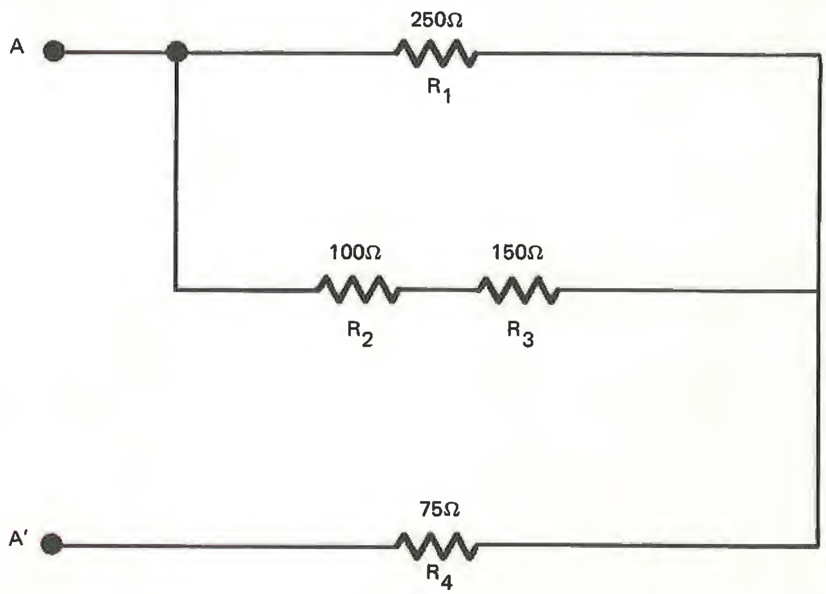


Fig. 8-4 The Experimental Circuit

3. Connect the source to points A and A' and set the source voltage to about 30 volts. Record this value as E_T in the data table.
4. Measure and record the voltage across each resistor (E_1 across R_1, E_2 across R_2, etc.)
5. Similarly, measure and record the current through each resistor.
6. Use the measured values of the individual component currents and voltages to determine the power dissipated by each resistor. Record the values as measured data.
7. Using the measured values of power and eq. 8.4, determine the total circuit power and record it as a measured value in the data table.
8. With the measured values of the individual resistors, compute the total effective circuit resistance and record it as a computed value.

Qty.	Measured Values	Computed Values
R_1		250 ohms
R_2		100 ohms
R_3		150 ohms
R_4		75 ohms
R_T		
E_T		≈ 30 volts
E_1		
E_2		
E_3		
E_4		
I_1		
I_2		
I_3		
I_4		
P_T		
P_1		
P_2		
P_3		
P_4		

Fig. 8-5 The Data Table

9. Use the measured values of E_T and the resistance to determine computed values for the individual component voltages, currents, and powers.

10. Take the sum of the computed values of power and record it as the computed value of total power.

ANALYSIS GUIDE. An important part of the objective of this experiment is to examine methods of analysis appropriate for use with series-parallel circuits. In examining your data, you should place particular emphasis on the extent of the validity of the analysis techniques used. Specifically, you should consider the extent of agreement between your computed values and the measured ones.

PROBLEMS

1. A certain voltage divider of the type shown in figure 8-3 has a source voltage of 250 volts. E_1 and E_2 are 190 volts and 90 volts respectively, while I_1, I_2, and I_3 are all 20mA. What are the values of R_1, R_2, and R_3?

2. In Problem 1, what total power is delivered to the loads? How much power must the source supply?

3. What are the values of the load resistances in Problem 1?

4. What is the total effective resistance connected across the source in Problem 1?

INTRODUCTION. In many practical cases, electrical and electronic circuits involve arrangements in which more than one source supplies current to a network. In this experiment we shall examine an analysis technique which is appropriate for use in such a case.

DISCUSSION. A good example of two sources sharing a common load is the charging circuit of an automobile. Figure 9-1 is a schematic representation of such a circuit.

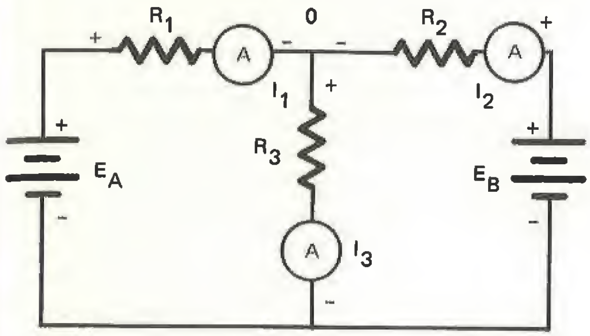


Fig. 9-1 A Multisource Network

We might think of E_A as the automobile battery and E_B as the charging device (generator or alternator circuit). R_1 and R_2 would then be the internal resistances of the sources while R_3 would be the load (headlights, etc.).

Perhaps the most common way to analyze the operation of such a circuit is to write *loop and nodal equations* using Kirchhoff's laws. To write a loop equation we first assume directions of flow for each of the currents I_1, I_2, and I_3. Then mark the polarities of the voltage drops across each element. (This has been done in figure 9-1.) We then select a closed loop, such as the one formed by E_A, R_1 and R_3. Since from Kirchhoff's voltage law the sum of the voltage drops around this loop must be equal to zero, we have:

$$E_1 + E_3 - E_A = 0$$

This equation may be readily arrived at by starting with any point and proceeding around the loop summing the voltage drops and rises. Each voltage is assigned the algebraic sign of the polarity on the input side of the particular element. Thus, E_1 and E_3 are positive while E_A is negative.

The loop equation may be transformed by observing that

$$E_1 = I_1R_1 \quad \text{and} \quad E_3 = I_3R_3$$

Making these substitutions renders

$$I_1R_1 + I_3R_3 - E_A = 0 \qquad \textbf{(9.1)}$$

In exactly similar manner, the equation for the loop including E_B, R_2, and R_3 will be:

$$I_2R_2 + I_3R_3 - E_B = 0 \qquad \textbf{(9.2)}$$

Since these two loops include all of the elements in the circuit, it is not necessary to write other loop equations. For purposes of illustration only, it is worthwhile to observe that there is a third loop formed by E_A, R_1, R_2, and E_B. The loop equation for this third loop would be:

$$I_1R_1 - I_2R_2 + E_B - E_A = 0$$

Inspection will reveal that this equation may also be arrived at by subtracting equation 9.2 from equation 9.1 and therefore does not contain any circuit information not already available from equation 9.1 and 9.2.

Because equations 9.1 and 9.2 contain three unknowns (I_1, I_2, and I_3), it is necessary to find a third independent equation in these three quantities. Such a third equation can be written for the *circuit node* (or junction) located at point 0. This equation will be:

$$I_1 + I_2 - I_3 = 0 \qquad \textbf{(9.3)}$$

The three current equations (9.1, 9.2, and 9.3) may now be solved simultaneously for the current values. With the currents known, any other circuit quantity may be found.

Since this technique is somewhat involved, it is perhaps worthwhile to work through a numerical example. Consider the circuit shown in figure 9-2.

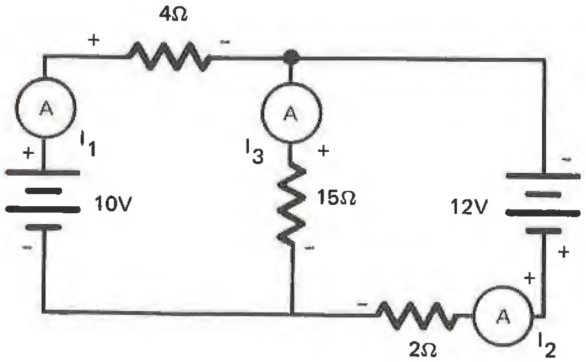


Fig. 9-2 A Typical Multisource Circuit

Current directions and polarities have been marked and the loop equations are:

$$4I_1 + 15I_3 - 10 = 0$$

and

$$2I_2 - 15I_3 - 12 = 0$$

The nodal equation in this case is

$$I_1 - I_2 - I_3 = 0$$

Solving the nodal equation for I_2 and substituting into the loop equation renders:

$$4I_1 + 15I_3 - 10 = 0$$

and

$$2I_1 - 17I_3 - 12 = 0$$

Multiplying the lower equation by 2 and then subtracting the two equations gives

$$49I_3 + 14 = 0$$

from which

$$I_3 = -0.29 \text{ amps}$$

Substituting this value back into one of the loop equations provides

$$I_1 = 3.57 \text{ amps}$$

and similarly,

$$I_2 = 3.86 \text{ amps}$$

The fact that I_3 is a negative number is worth noting. The significance of the negative sign is that I_3 actually flows in the direction opposite to that shown in figure 9-2. Similarly, the positive signs (now shown) of I_1 and I_2 indicate that they do indeed flow in directions indicated in the figure.

With the values of the currents determined, the individual element voltage drops, power, etc., may be computed using the customary methods.

MATERIALS

2 Variable DC power supplies
1 Multimeter
1 75-ohm resistor
1 250-ohm resistor
1 150-ohm resistor
1 100-ohm resistor

PROCEDURE

1. Measure the ohmic value of each of the resistors and record the values in the data table (figure 9-4).
2. Construct the circuit shown in figure 9-3.

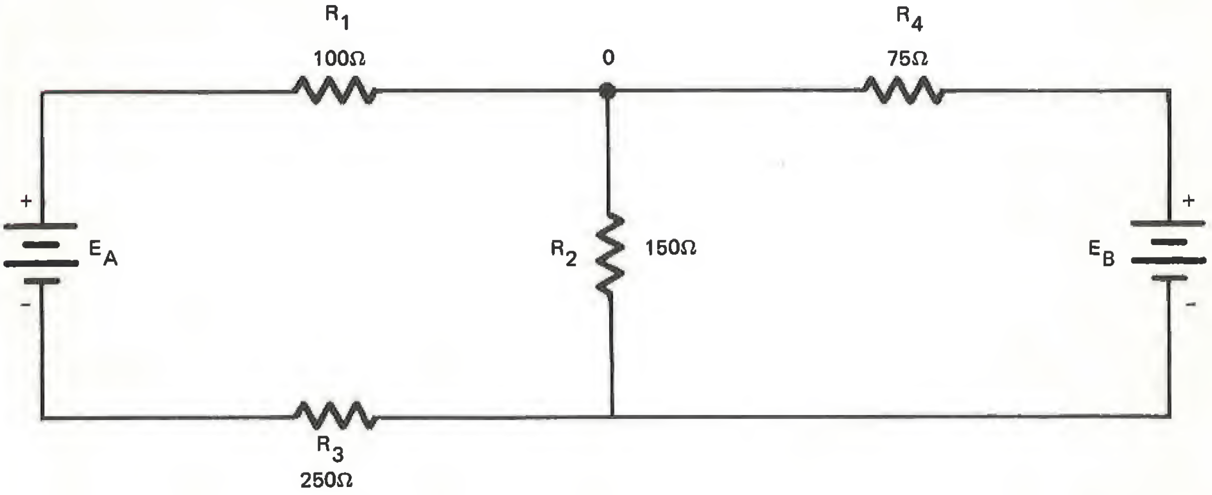


Fig. 9-3 The Experimental Network

3. As accurately as possible, set the value of E_A to 30.0 volts and E_B to 24.0 volts. Use the multimeter to measure these voltages. *Do not rely on the voltmeter contained in the power supplies.*
4. Measure the current flow through each resistor and record it in the data table. (Use I_1 for the current through R_1; I_2 through R_2; etc.).
5. In similar manner, measure and record the voltage drop across each resistor.
6. Disconnect both sources and replace source E_B with a short circuit. Measure the total effective circuit resistance (R_A) from the E_A end.
7. Similarly, short circuit the E_A end and measure the resistance (R_B) from the E_B end.
8. Using the measured values of individual element voltage and current, compute the power dissipation of each element and record the values as measured data.
9. With the measured values of source voltage and current, compute the power supplied by each source (P_A and P_B). Record these values as measured quantities.

Qty.	Measured Value	Computed Value
E_A	30 volts	30 volts
E_B	24 volts	24 volts
R_1		100 ohms
R_2		150 ohms
R_3		250 ohms
R_4		75 ohms
R_A		
R_B		
I_1		
I_2		
I_3		
I_4		
E_1		
E_2		
E_3		
E_4		
P_1		
P_2		
P_3		
P_4		
P_A		
P_B		
P_T		

Fig. 9-4 The Data Table

10. Determine the total network power dissipation (P_T) by taking the sum of all of the individual component powers. Record the result as a measured value.

11. Use the specified values of resistance and voltage. Write loop and nodal equations for the circuit currents and record the results as computed data.

12. With the currents from step 11 and the specified values of resistance, compute the values of the individual element voltage and power.

13. Assuming E_B to be shorted, compute the value of R_A using the specified values of the resistors.

14. Similarly, compute R_B.

15. Compute the total source power (P_T) by taking the sum of P_A and P_B.

ANALYSIS GUIDE. In evaluating the results of this experiment, you should in general compare the measured values to the computed ones. In particular, consider the significance of R_A and R_B. Were these the values of load applied to each of the sources? Did the total circuit dissipation found in step 10 agree with the source power in step 15?

PROBLEMS

1. Discuss the conditions which must exist in figure 9-1 for the value of I_2 to be equal to zero.

2. Write loop equations and solve for the values of I_1, I_2, and I_3 in the circuit shown in figure 9-5.

3. Repeat problem 2 with the positions of the 10kΩ and 20kΩ resistors reversed.

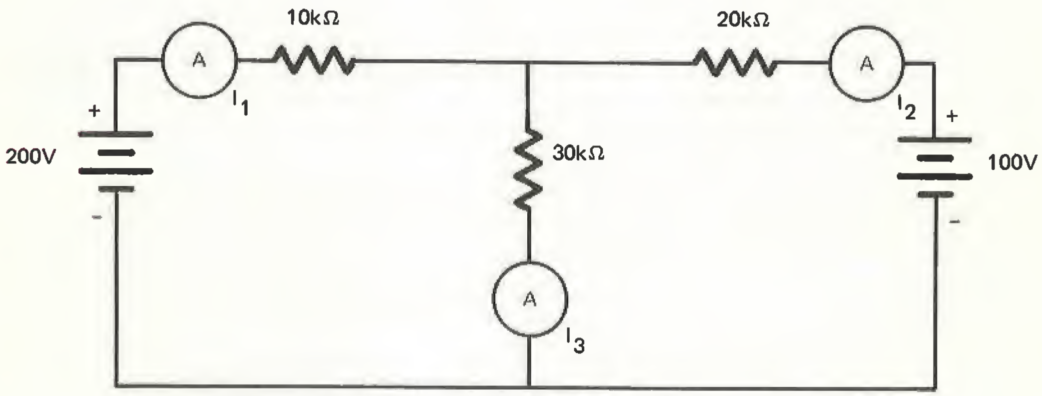


Fig. 9-5 Circuit for Problem 2

experiment 10 NETWORK THEOREMS

INTRODUCTION. In electricity, theorems are little more than formalized statements of useful techniques of circuit analysis. It is, of course, possible to establish these theorems, either theoretically and empirically. In this experiment, however, we shall be primarily interested in examining the utilization of the theorems rather than their proof.

DISCUSSION. The four most commonly encountered network theorems are:

1. *Thévenin's Theorem.* Any two-terminal network containing only linear resistances and sources may be represented by a single linear resistor and *constant voltage* source connected in series.
2. *Norton's Theorem.* Any two-terminal network containing only linear resistances and sources may be represented by a single linear resistor and *constant current* source connected in parallel.
3. *Superposition Theorem.* The current flow caused by several sources in a circuit branch is equal to the algebraic *sum* of the component currents from each source taken separately in turn while the remaining sources are replaced by their respective internal resistances.
4. *Delta-Wye Transformation.* Any three-terminal network (sometimes considered a four-terminal network) which is in a delta configuration can be converted to a corresponding wye network and vice-versa.

Each one of the network theorems requires some explanation before it may be applied in circuit analysis. Perhaps the best way to explain their uses is to work through a circuit analysis example with each theorem in turn. It should be understood that in a particular problem one theorem may be more appropriate than another. In the following examples, the purpose is to illustrate and explain each theorem, whether it is the most appropriate one for the problem or not.

The circuit for the first examples we shall consider is given in figure 10-1. Let us determine the current (I_L) through the load resistor (R_L) using each of the circuit theorems listed above.

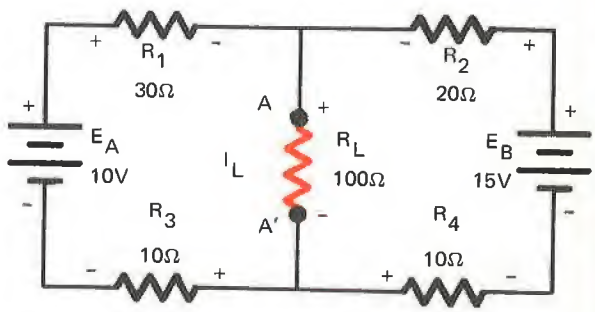


Fig. 10-1 Circuit Used in the Example

1. *Thévenin's Theorem.* To apply Thévenin's Theorem, we disconnect the load at points A and A′ and compute the open circuit voltage between the points as shown in figure 10-2.

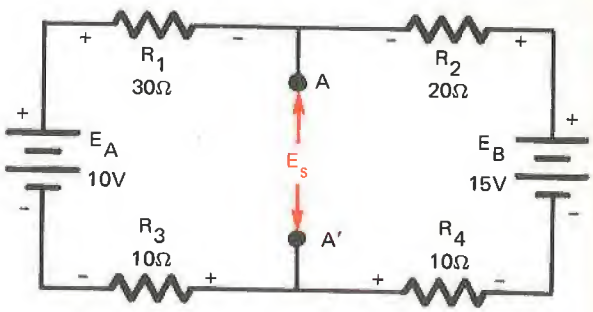


Fig. 10-2 The Circuit with the Load Removed

The total resistance in the circuit will be

$$R_T = R_1 + R_2 + R_3 + R_4 =$$

$$30 + 20 + 10 + 10 = 70\Omega$$

The total current will be

$$I_T = \frac{E_B - E_A}{R_T} = \frac{5}{70} = 0.0714A$$

Finally, E_s will be

$$E_s = E_B - I_T R_2 - I_T R_4 =$$

$$15 - 1.428 - 0.714 = 12.858 \text{ volts}$$

Next we determine the circuit resistance across points A and A′ by replacing E_A and E_B with short circuits as shown in figure 10-3.

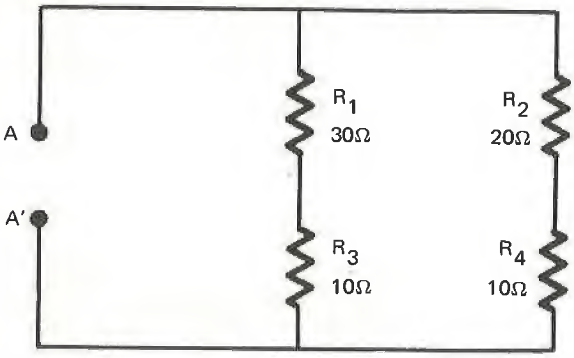


Fig. 10-3 Circuit for Determining R_s

The value of R_s will be

$$R_s = \frac{(R_1 + R_3)(R_2 + R_4)}{R_1 + R_2 + R_3 + R_4} =$$

$$\frac{(30 + 10)(20 + 10)}{30 + 20 + 10 + 10} = \frac{(40)(30)}{70} = 17.15\Omega$$

The Thévenized equivalent circuit of figure 10-1 can now be drawn as in figure 10-4.

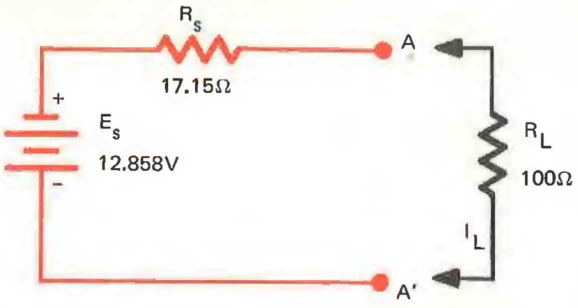


Fig. 10-4 The Thévenized Circuit

From this circuit we can compute I_L by

$$I_L = \frac{E_s}{R_s + R_L} = \frac{12.858}{117.15} = 0.1098 \text{ A}$$

2. *Norton's Theorem.* As above, we must develop an equivalent circuit using Norton's theorem before we can determine I_L. To develop the equivalent circuit one proceeds just as with Thévenin's Theorem until the values of E_s and R_s are found. Then the value of Norton's constant current generator is determined by

$$I_s = \frac{E_s}{R_s} = \frac{12.858}{17.15} = 0.749 \text{ A}$$

The Norton's equivalent circuit of figure 10-5 may now be drawn.

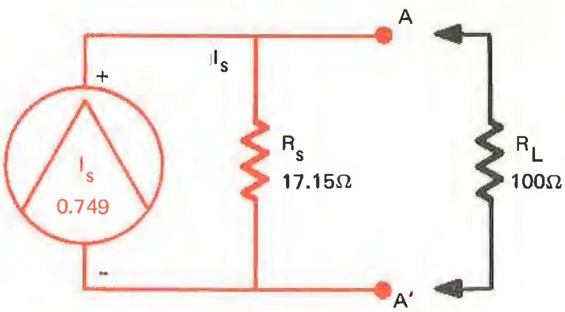


Fig. 10-5 The Norton's Equivalent Circuit

The value of the load current may now be determined.

$$I_l = I_s \frac{R_s}{R_s + R_L} = 0.749 \frac{17.15}{117.15}$$

$$= 0.1098 \text{ A}$$

3. *Superposition Theorem.* In applying the superposition theorem, we eliminate all but one source at a time by replacing all other sources by their internal resistance. Let us consider that R_1 in figure 10-1 is the internal resistance of source E_A while R_2 is the internal resistance of source E_B. If we eliminate source E_B (by short circuiting it), the circuit is as shown in figure 10-6a.

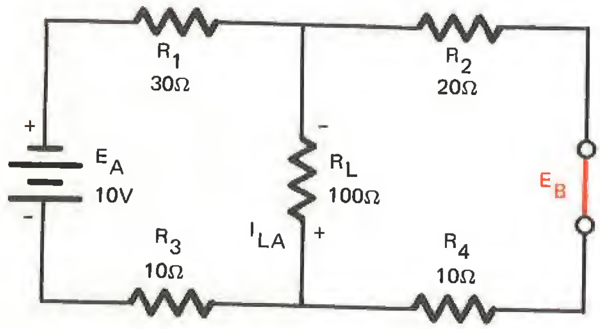


Fig. 10-6a Circuit With E_B Shorted

The load current component caused by E_A (I_{LA}) can now be found by conventional circuit analysis methods.

$$R_T = R_1 + R_3 + \frac{R_L (R_2 + R_4)}{R_L + R_2 + R_4}$$

$$= 30 + 10 = \frac{100\,(20 + 10)}{130} = 63.05\Omega$$

$$I_T = \frac{E_A}{R_T} = \frac{10}{63.05} = 0.1587 \text{ A}$$

$$I_{LA} = I_T \frac{R_2 + R_4}{R_L + R_2 + R_4}$$

$$= 0.1587 \frac{20 + 10}{130} = 0.0366 \text{ A}$$

If source E_B is restored to the circuit and E_A is removed, then the circuit appears as in figure 10-6b.

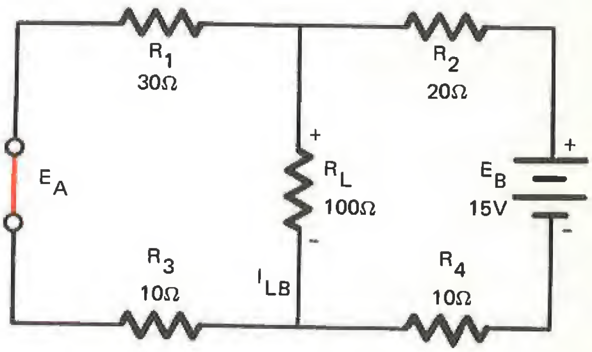


Fig. 10-6b Circuit With E_A Shorted

The component of the load current due to E_A (I_{LB}) may now be determined as before

$$R_T = R_2 + R_4 + \frac{R_L (R_1 + R_3)}{R_L + R_1 + R_3} = 20 + 10$$

$$+ \frac{100\,(30 + 10)}{140} = 58.6\Omega$$

$$I_T = \frac{E_B}{R_T} = \frac{15}{58.6} = 0.256 \text{ A}$$

$$I_{LB} = I_T \frac{R_1 + R_3}{R_L + R_1 + R_3} = 0.256$$

$$\frac{40}{140} = 0.0732 \text{ A}$$

The net current through R_L may now be computed by

$$I_L = I_{LA} + I_{LB} = 0.366 + 0.732 = 0.1098 \text{ A}$$

In general electricity, Thévenin's is perhaps the most frequently encountered of the three theorems. In electronics, however, both Thévenin's and Norton's Theorems are extensively used.

4. *Delta-Wye Networks.* Another useful transformation technique is that of converting a delta network into its equivalent wye network and vice-versa. Let us consider this derivation of the transformation equations with the help of figure 10-7.

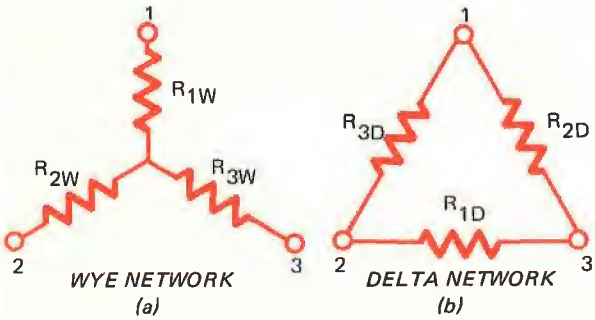


Fig. 10-7 Delta-Wye Networks

First, let us consider terminals 1 and 2 of each network. If the circuits are to be equivalent, then,

$$R_{12W} = R_{12D}$$

In diagram a (ignoring terminal 3), we find,

$$R_{12W} = R_{1W} + R_{2W}$$

and in diagram b,

$$R_{12D} = \frac{R_{3D}(R_{1D} + R_{2D})}{R_{3D} + R_{1D} + R_{2D}}$$

and assuming R_{12W} is to be equal to R_{12D}, then by substitution we find,

$$R_{1W} + R_{2W} = \frac{R_{3D}(R_{1D} + R_{2D})}{R_{3D} + R_{1D} + R_{2D}}$$

Let us next consider terminals 2 and 3, ignoring terminal 1:

$$R_{23W} = R_{2W} + R_{3W}$$

$$\text{and, } R_{23W} = \frac{R_{1D}(R_{2D} + R_{3D})}{R_{1D} + R_{2D} + R_{3D}}$$

Let us next consider terminals 1 and 3, ignoring terminal 2:

$$R_{13W} = R_{1W} + R_{3W}$$

$$\text{and, } R_{13D} = \frac{R_{2D}(R_{1D} + R_{3D})}{R_{1D} + R_{2D} + R_{3D}}$$

Subtracting the R_{23} equivalent from R_{12} equivalent, we obtain,

$$R_{12W} - R_{23W} = (R_{1W} + R_{2W})\ (R_{2W} + R_{3W})$$

$$= \frac{R_{3D}(R_{1D} + R_{2D})}{R_{3D} + R_{1D} + R_{2D}} = \frac{R_{1D}(R_{2D} + R_{3D})}{R_{1D} + R_{2D} + R_{3D}}$$

So,

$$R_{1W} - R_{3W} = \frac{R_{2D}R_{3D} - R_{1D}R_{2D}}{R_{1D} + R_{2D} + R_{3D}}$$

And by adding the R_{13} equivalent to the above, we obtain,

$$2R_{1W} = \frac{2R_{2D}R_{3D}}{R_{1D} + R_{2D} + R_{3D}}$$

And,

$$R_{1W} = \frac{R_{2D}R_{3D}}{R_{1D} + R_{2D} + R_{3D}}$$

Using the same approach, we then can determine the delta circuit equivalents for R_{2W} and R_{3W}:

$$R_{2W} = \frac{R_{1D}R_{3D}}{R_{1D} + R_{2D} + R_{3D}}$$

$$R_{3W} = \frac{R_{1D}R_{2D}}{R_{1D} + R_{2D} + R_{3D}}$$

The wye-to-delta transformation equations can, by similar technique, be determined to be:

$$R_{1D} = \frac{R_{1W}R_{2W} + R_{2W}R_{3W} + R_{1W}R_{3W}}{R_{1W}}$$

$$R_{2D} = \frac{R_{1W}R_{2W} + R_{2W}R_{3W} + R_{1W}R_{3W}}{R_{2W}}$$

$$R_{3D} = \frac{R_{1W}R_{2W} + R_{2W}R_{3W} + R_{1W}R_{3W}}{R_{3W}}$$

These above equations can also be very useful in the analysis of various circuits.

MATERIALS

- 2 Multimeters (VOM)
- 2 Variable DC power supplies
- 1 75 ohm resistor
- 1 100 ohm resistor
- 1 150 ohm resistor

PROCEDURE

1. Measure and record in the data table (figure 10-11) the values of the three resistors.
2. Assemble the circuit shown in figure 10-8.
3. Adjust the source voltages E_A and E_B to about 25 and 15 volts respectively. Record the values of the voltages in the data table.

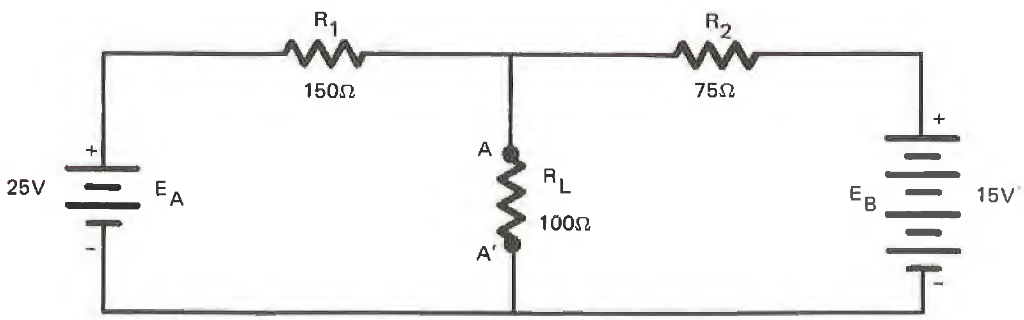


Fig. 10-8 The Test Circuit

4. Measure and record the value of the current (I_L) through the 100 ohm load.
5. Measure and record the value of the voltage (E_L) across the 100 ohm load.
6. Disconnect the 100 ohm load resistor at points A and A′. Measure the open circuit voltage between the points (E_s) and record it in the data table.
7. Using Thévenin's Theorem, compute the value of the effective source resistance (R_s) and record it in the data table.
8. Disassemble the test circuit and connect the circuit shown in figure 10-9. The resistance R_s *can* be formed using the values of resistors specified in the materials list. Set the power supply to the value of E_s measured in step 6. Leave the voltmeter connected to monitor the value of E_s. Do not connect the load resistor in this step.
9. Connect the load and measure the value of the load voltage and current. Be sure that the value of E_s is maintained during this step.
10. Compute and record the value of the Norton's equivalent current (I_s).
11. Disassemble the Thévenized circuit and construct the Nortonized circuit shown in figure 10-10. Do not connect the load resistor in this step.

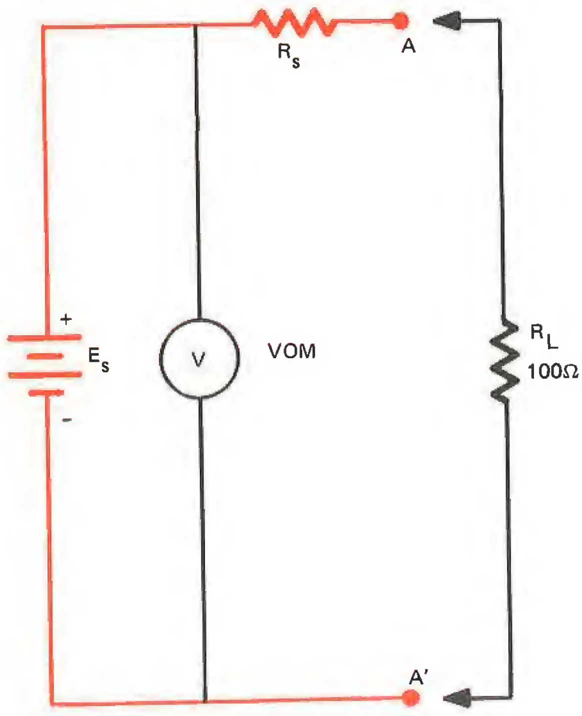


Fig. 10-9 The Thévenized Circuit

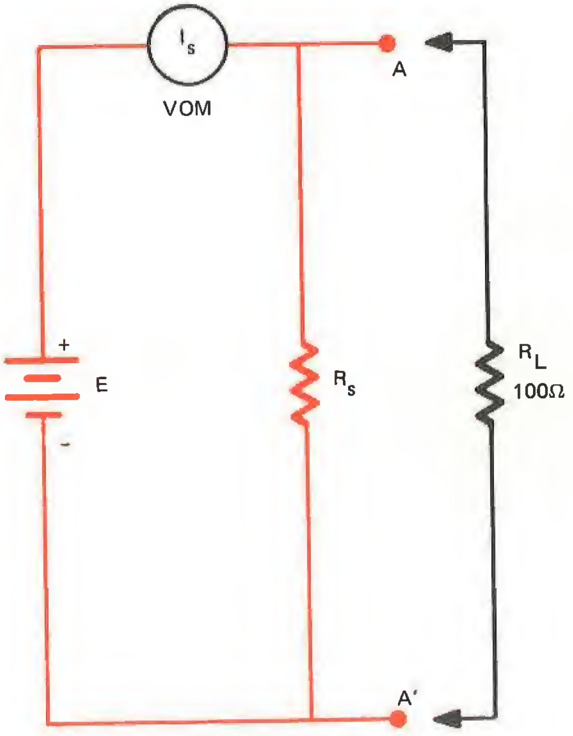


Fig. 10-10 The Nortonized Circuit

12. Adjust the voltage E until the current meter reads the value of I_s.
13. Connect the load and measure the values of I_L and E_L. Be very sure that the source current I_s is held to the value computed in step 10.
14. Disassemble the Nortonized equivalent circuit and reassemble the original test circuit.
15. Set the voltages E_A and E_B to the levels measured in step 3.
16. Remove the E_B source from the network and replace it with a short circuit. Measure and record the load current (I_{LA}) due to the E_A source alone.
17. Replace the E_B source, remove the E_A source, and determine the load current (I_{LB}) due to source E_B alone.
18. Determine the net load current from the values of the component currents. Record this value as measured data.
19. Using the appropriate technique in each case, compute the remaining data table values based on the specified values of R_1, R_2, R_L, E_A, and E_B.

Qty	R_1	R_2	R_L	E_A	E_B	I_L	E_L	E_s
Computed	150	75	100	25V	15V			
Measured								

	Thévenized Circuit			Nortonized Circuit			Superposition		
Qty	R_s	E_L	I_L	I_s	E_L	I_L	I_{LA}	I_{LB}	I_L
Computed									
Measured									

Fig. 10-11 The Data Table

ANALYSIS GUIDE. The purpose of this exercise has been to demonstrate use and effectiveness of the three network theorems. In considering the significance of your data, you should examine the extent of agreement between the various values of E_L and I_L.

PROBLEMS

1. In the network shown in figure 10-12, what would be the open circuit voltage and Thévenized resistance if R_4 were considered the load?

2. Determine the values of I_s and R_s of the Norton's equivalent circuit for figure 10-12 if R_6 were the load.

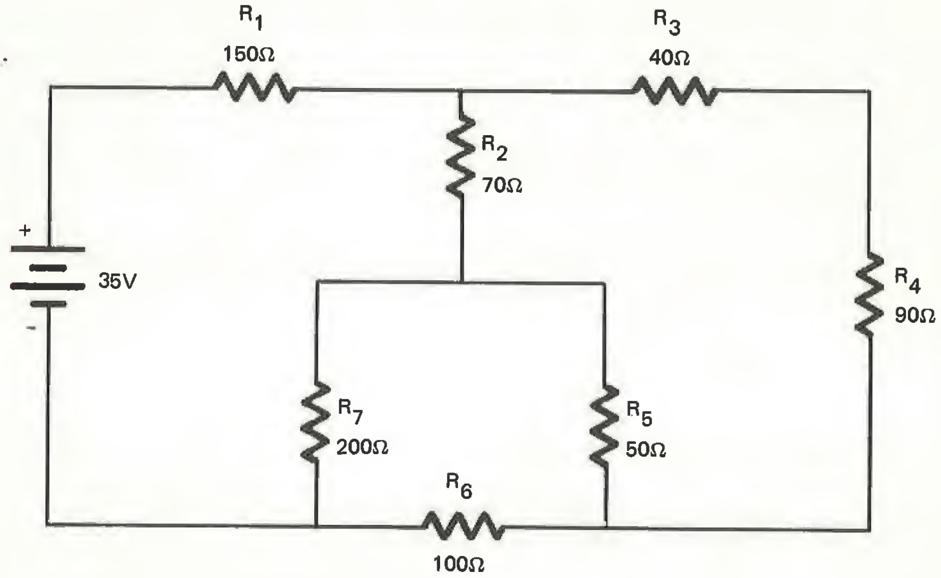


Fig. 10-12 Circuit for Problem 1

experiment 11 WHEATSTONE BRIDGES

INTRODUCTION. The *wheatstone bridge* circuit is very widely used in measurement work. In this experiment we shall examine the operation of the bridge circuit under balanced and unbalanced conditions.

DISCUSSION. A wheatstone bridge circuit is composed of two *delta* networks having one common side. Figure 11-1 shows a typical bridge network. The center resistance (R_L) is usually a current meter. When the bridge is in the condition called *balance,* the current

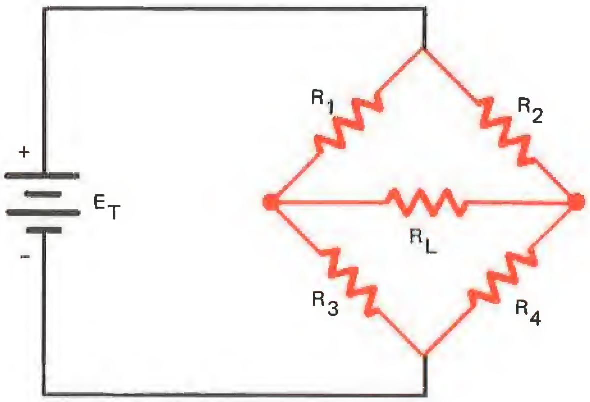


Fig. 11-1 A Wheatstone Bridge Circuit

through R_L (the current meter) is zero. For this condition to exist, the voltage drop across R_L must be zero. Or in other words, the potentials at each end of R_L must equal each other. This will only occur when the voltage (E_3) across R_3 is equal to the voltage (E_4) across R_4. With zero current flow through R_L, the values of E_3 and E_4 may be found by voltage divider action to be

$$E_3 = E_T \frac{R_3}{R_1 + R_3} \quad \text{and}$$

$$E_4 = E_T \frac{R_4}{R_2 + R_4} \tag{11.1}$$

Therefore, at balance we have

$$E_3 = E_4$$

or

$$E_T \frac{R_3}{R_1 + R_3} = E_T \frac{R_4}{R_2 + R_4}$$

from which

$$\frac{R_3}{R_1 + R_3} = \frac{R_4}{R_2 + R_4}$$

Taking the reciprocal of each side gives

$$\frac{R_1 + R_3}{R_3} = \frac{R_2 + R_4}{R_4}$$

which may be rewritten as

$$\frac{R_1}{R_3} + 1 = \frac{R_2}{R_4} + 1$$

and finally subtracting 1 from each side provides

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \tag{11.2}$$

This equation gives us the condition that will result in a balanced bridge.

When the bridge is not balanced, there will be a current flow through R_L (the current meter). The amount of this current can be determined by Thévenizing the bridge as shown in figure 11-2.

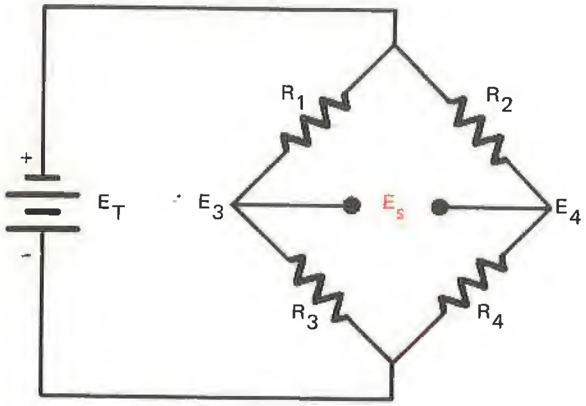


Fig. 11-2 A Bridge With the Load Removed

The open circuit voltage (E_s) at the load terminals will be

$$E_s = E_3 - E_4$$

Substituting the relationships from equation 11.1 gives us

$$E_s = E_T \frac{R_3}{R_1 + R_3} - E_T \frac{R_4}{R_2 + R_4}$$

which may be written as

$$E_s = E_T \left(\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right)$$

And the source resistance R_s of the Thévenized circuit can be found by replacing E_T with a short circuit and redrawing the circuit as in figure 11-3.

R_s may then be seen to be

$$R_s = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

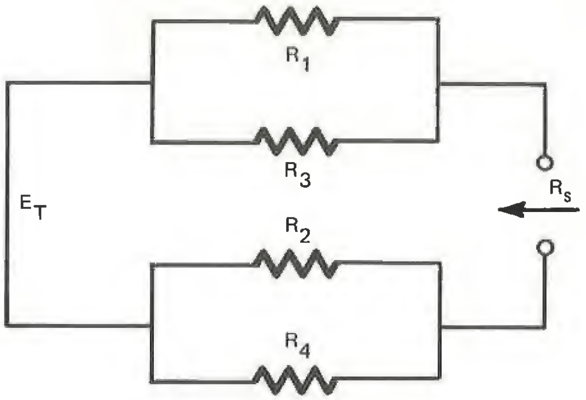


Fig. 11-3 Determining R_s

The Thévenization of the bridge may now be completed and drawn as shown in figure 11-4.

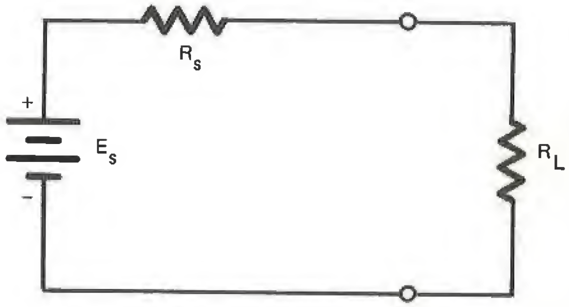


Fig. 11-4 The Thévenized Bridge

The current through the load (I_L) can now be determined by

$$I_L = \frac{E_s}{R_s + R_L}$$

In many practical applications, the value of I_L is so small (R_L is very large) that it may be neglected. In such an application the circuit diagram of the bridge becomes that shown in figure 11-2.

The usual application of the bridge as a measuring device is implemented by making three of the legs fixed resistances (R_1, R_2, R_3 for instance). The remaining resistor (R_4) is a variable used to sense the quantity to be measured (temperature, force, or pressure, for example). Since R_1 and R_3 are fixed values, E_3 will also be a fixed value if I_L is very small. When this is the case, E_s will be directly proportional to the amount (ΔR_4) by which R_4 varies from the balanced value. From the diagram we see that

$$E_4 + \Delta E_4 = E_T \frac{R_4 + \Delta R_4}{R_2 + R_4 + \Delta R_4}$$

will be the new value of voltage at the junction of R_2 and R_4 after R_4 changes by the amount ΔR_4. We may rewrite this in the form

$$\Delta E_4 = E_T \frac{R_4 + \Delta R_4}{R_2 + R_4 + \Delta R_4} - E_4$$

Then substituting equation 11-1 for E_4 we have

$$\left[\Delta E_4 = E_T \frac{R_4 + \Delta R_4}{R_2 + R_4 + \Delta R_4} - \frac{R_4}{R_2 + R_4} \right]$$

Finding the common denominator and subtracting gives us

$$\Delta E_4 = E_T \frac{R_2 \Delta R_4}{(R_2 + R_4 + \Delta R_4)(R_2 + R_4)}$$

If R_4 is very small compared to $R_2 + R_4$, we can write this as

$$\Delta E_4 = E_T \frac{R_2 \Delta R_4}{(R_2 + R_4)^2}$$

Now since E_3 is constant, it happens that

$$E_s = \Delta E_4$$

Therefore,

$$\boxed{E_s \approx E_T \frac{R_2 \Delta R_4}{(R_2 + R_4)^2}}$$

or in other words, the output voltage (E_s) of the bridge is directly proportional to the change in R_4.

For the specific circuit of figure 11-2 and when E_s is defined as $E_s = E_3 - E_4$, then E_s will increase with a decrease in R_4 and likewise E_s will decrease for an increase in R_4.

MATERIALS

- 1 Variable DC power supply
- 1 Multimeter
- 1 75 ohm resistor
- 1 100 ohm resistor
- 1 150 ohm resistor
- 1 250 ohm resistor
- 1 Resistance decade box
- 2 Sheets of linear graph paper

PROCEDURE

1. Measure and record the ohmic value of each of the specified resistors.
2. Assemble the circuit shown in figure 11-5. Use the resistance decade box for R_L.

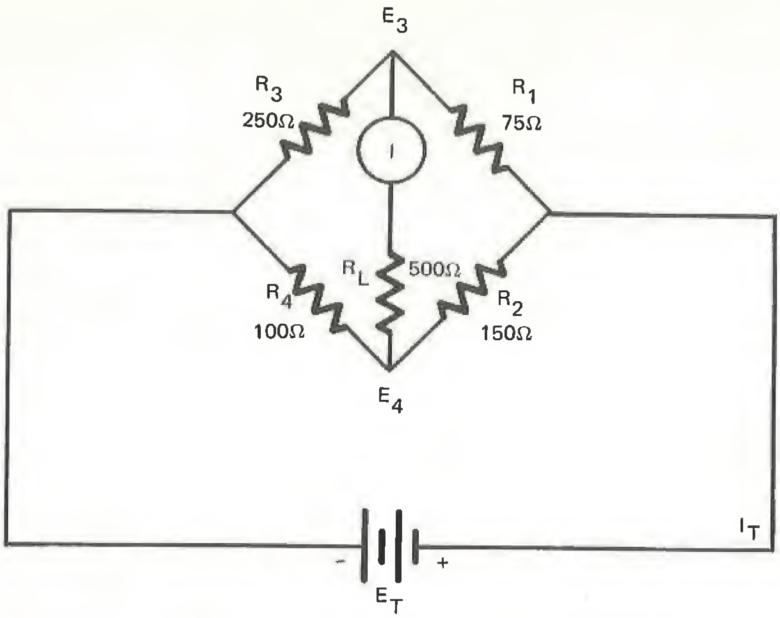


Fig. 11-5 The Test Circuit

3. Set the level of E_T to about 15 volts and record the value in the data table.
4. Measure and record the load current I_L.
5. Remove the meter from the circuit, then measure and record each of the currents I_1 (through R_1), I_2 (through R_2), I_3 (through R_3), I_4 (through R_4), and I_T.
6. Disconnect the load and measure the open circuit load voltage E_s.
7. Remove the 100 ohm resistor (R_4) and replace it with the resistance decade box.
8. Connect the multimeter to measure the open circuit load voltage.
9. Measure and record the value of the open circuit load voltage (E_s) for R_4 values of 100, 200, 300, 400, 500, 600, 700, 800, 900, and 1000 ohms.
10. Connect the 100 ohm resistor as the load. Measure and record the voltage (E_L) for each of the values of R_4 given in step 9.
11. Using Thévenin's Theorem, calculate the value of I_L with the circuit as it was in step 4.
12. Write Kirchhoff Law equations and solve for each of the currents measured in step 5.
13. Calculate the percent difference between each pair of computed and measured values.
14. On a sheet of linear graph paper, plot the value of R_4 (along the X axis) versus the measured values of E_s (along the Y axis).
15. Similarly, plot the value of R_4 versus E_L on a second sheet of graph paper.

Qty	Measured Values	Computed Values	Percent Difference	Data for Load Voltage Plot		
R_1		75Ω		R_4 Ohms	E_s	E_L
R_2		150Ω		100		
R_3		250Ω		200		
R_4		100Ω		300		
E_T		15V		400		
I_L				500		
I_1				600		
I_2				700		
I_3				800		
I_4				900		
I_T				1000		

Fig. 11-6 The Data Table

ANALYSIS GUIDE. In the analysis of these data you should consider the extent to which the open circuit load voltage was directly proportional to the variation in value of R_4. Did the 100 ohm load change the extent of proportionality? If so, why?

PROBLEMS

1. What would be the total resistance of the circuit on the supply (E_T) if the bridge in the experiment were balanced?
2. What would be the Thévenized resistance of the bridge at balance?
3. Why are the values found in problem 1 and problem 2 different?
4. Assume that the bridge in the experiment is to be used to measure the temperature inside an oven. If the bridge is balanced at 400°F and each volt of E_s in step 9 is equivalent to a change of 10°F, what was the maximum temperature reached in the experiment?

experiment 12 METER CIRCUITS

INTRODUCTION. Basic electric meters play a vital role in any electrical laboratory situation. In this experiment we shall examine the internal circuitry of an elementary *ammeter, voltmeter,* and *ohmmeter.*

DISCUSSION. The most commonly encountered type of instrument movement is the *permanent-magnet* moving-coil system. A simplified sketch of this type of instrument is shown in figure 12-1.

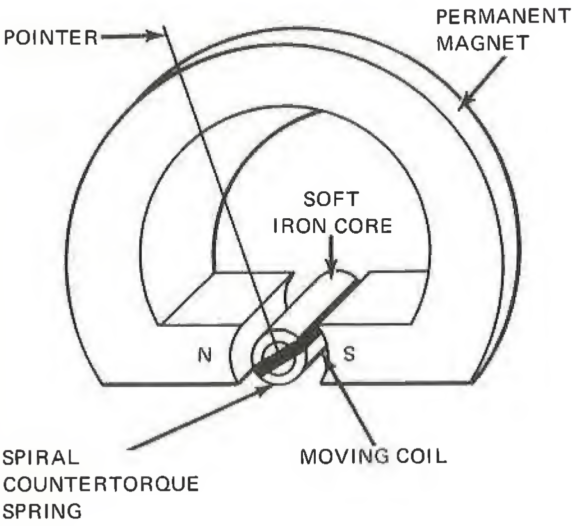


Fig. 12-1 A Permanent-Magnet Moving-Coil Instrument

As a DC current is passed through the moving coil, it will tend to rotate about the soft iron core due to simple motor action. The force acting on each wire of the coil will be

$$F = B\ell I_m$$

where B is the flux density established by the permanent magnet, ℓ is the coil length and I_m is the current.

Since this force acts on both sides of the coil simultaneously, the total torque acting on the coil will be

$$T_1 = Fd = B\ell I_m Nd$$

N is the number of turns on the coil and d is the diameter of the coil. The important point is that B, ℓ, N, and d are constants in a given meter. The torque is therefore directly proportional to the current:

$$T_1 = K_1 I_m$$

This torque is directly opposed by the countertorque of the spiral spring. A spring of this type generates a countertorque which is directly proportional to angular distance θ through which its end moves from rest. That is,

$$T_2 = K_2 \theta$$

The overall result of the two opposing torques is that when current flows in the moving coil, the pointer swings up scale until the coil torque just equals the spiral spring countertorque. The meter pointer comes to rest when

$$T_2 = T_1$$

or

$$\theta = \frac{K_1}{K_2} I_m$$

The meter reading, θ, is therefore directly proportional to the coil current. This situation allows us to build a meter with linear scale markings.

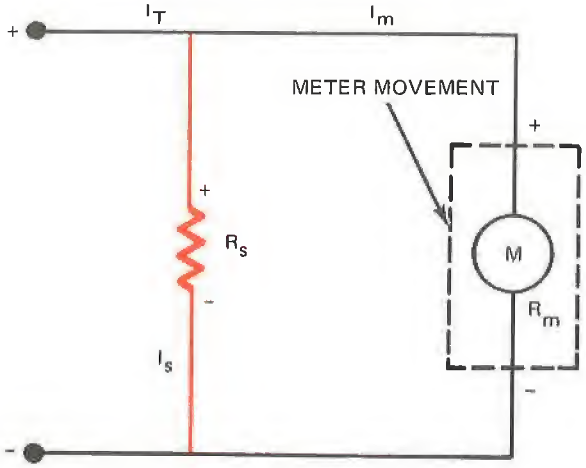


Fig. 12-2 A Shunted Instrument

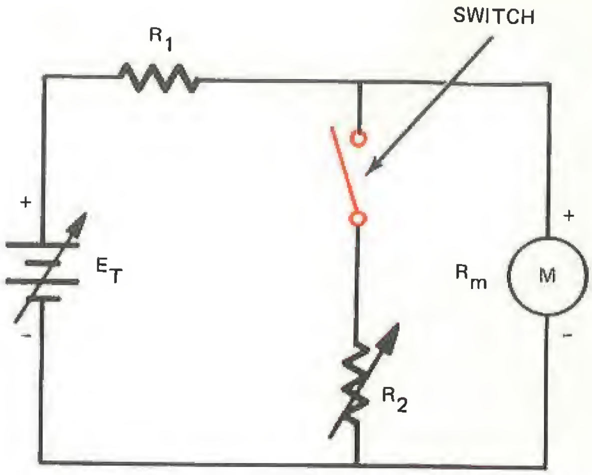


Fig. 12-3 Determining the Value of R_m

In many practical cases, the amount of current to be measured is large enough to cause the torques to come to balance beyond (off scale) the range of the scale marking. In such an event, it is reasonable to consider the possibility of diverting a portion of the current around the meter movement. Figure 12-2 shows a circuit with the meter shunted by a fixed resistor.

Applying the current division principle, we see that the current through the meter movement will be

$$I_m = I_T \frac{R_s}{R_s + R_m} \qquad (12.1)$$

In other words, the current through the instrument will be directly proportional to the total current if the values of R_s and R_m are constant (linear). The scale of the meter could, therefore, be marked to indicate values of I_T directly.

If the value of R_m is known, values of R_s can be calculated to provide any desired current measurement range greater than the basic range of the movement.

Figure 12-3 shows a circuit for determining the approximate value of R_m for a typical instrument. The value of the series resistor is selected so that a voltage within the adjustable range of the source will cause full scale current to flow through the meter. For example, if E_T is adjustable from zero to twenty volts and the meter has a zero to 1 mA range, we might choose R_1 as

$$R_1 = \frac{E}{I}$$

$$= \frac{10}{0.001}$$

$$= 10\text{k ohms}$$

The source is set for a full scale reading on the meter. The switch is now closed and R_2 is adjusted until the meter reads exactly one-half the full scale value. The value of R_2 is now equal to R_m.

With R_m known, we may now find the value of a shunt resistance to provide any desired full scale current reading. Equation 12.1 may be solved for R_s as follows:

$$I_m = I_T \frac{R_s}{R_s + R_m} \qquad (12.1)$$

First multiply both sides of equation 12.1 by $(R_s + R_m)$:

$$I_m R_s + I_m R_m = I_T R_s$$

Then collect R_s terms on the left:

$$I_T R_s - I_m R_s = I_m R_m$$

Now factor the R_s terms,

$$R_s (I_T - I_m) = I_m R_m$$

and divide by $(I_T - I_m)$:

$$\boxed{R_s = R_m \frac{I_m}{I_T - I_m}} \qquad (12.2)$$

Let us illustrate how the range of a current meter may be changed by assuming that we wish to use a 1 mA meter to measure currents up to 100 mA. Further, suppose that the value of R_m has been measured and found to be 50 ohms. The required shunt resistance will be

$$R_s = R_m \frac{I_m}{I_T - I_m} = (50) \frac{0.001}{0.1 - 0.001}$$

$$= 50\ (0.011) = 0.55 \text{ ohms}$$

It is, of course, possible to construct an instrument having several ranges by using a range selector switch and appropriate values of shunts.

The basic permanent-magnet moving-coil instrument may also be used as a voltmeter. We know from Ohm's Law that the voltage across the meter will be

$$E_m = I_m R_m$$

Since the meter voltage is directly proportional to the current and the current in turn is directly proportional to the meter deflection, we may conclude that the deflection will be directly proportional to the meter voltage.

If we wish to measure values of voltage greater than E_m at full scale, it is only necessary to connect the meter in series with a multiplier resistance as shown in figure 12-4.

The value of R_v required to change the full scale indication from E_m to E_T may be found by observing that

$$R_v + R_m = \frac{E_T}{I_m}$$

or

$$\boxed{R_v = \frac{E_T}{I_m} - R_m} \qquad (12.3)$$

To illustrate, let us presume that we wish to use the meter in the last example (I_m = 1 mA, R_m = 50Ω) to measure 25 volts full scale. The required value of R_v will be

$$R_v = \frac{E_T}{I_m} - R_m = \frac{25}{0.001} - 50$$

$$= 25{,}000 - 50 = 24{,}950\Omega$$

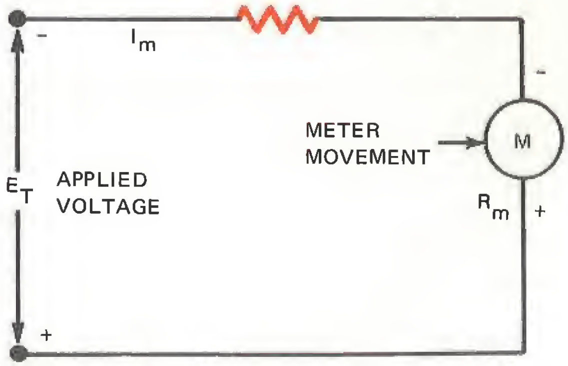


Fig. 12-4 A Voltmeter Circuit

The *sensitivity* (s) of a voltmeter is defined as the ratio of the meter resistance R_m to the full scale voltage E_m. That is

$$s = \frac{R_m}{E_m}$$

In the above example, E_m is 50 mV. The sensitivity of the instrument is then

$$s = \frac{R_m}{E_m} = \frac{50}{0.05} = 1000 \text{ ohms per volt}$$

We can observe that $I_m = E_m/R_m$, or in other words, $I_m = 1/s$. Equation 12.3 may therefore be rewritten as

$$R_v = sE_T - R_m \qquad \textbf{(12.4)}$$

The value of R_v in the example above could have been determined by

$$R_v = sE_T - R_m = 1000 \times 25 - 50$$

$$= 25{,}000 - 5 = 24{,}950\Omega$$

As in the case of the ammeter, a multi-range voltmeter may be constructed using a range selector switch and appropriate multiplier resistors.

If a source is included in the meter circuit as shown in figure 12-5, the instrument may be used to measure resistance.

To set up the instrument, one short circuits the test points A and A′ and adjusts R_o for full scale deflection. This reading (full scale) corresponds to a resistance (R_x) measurement of zero ohms. The value of R_o which produces this result is, of course,

$$R_o = \frac{E_b}{I_m} - R_m \qquad \textbf{(12.5)}$$

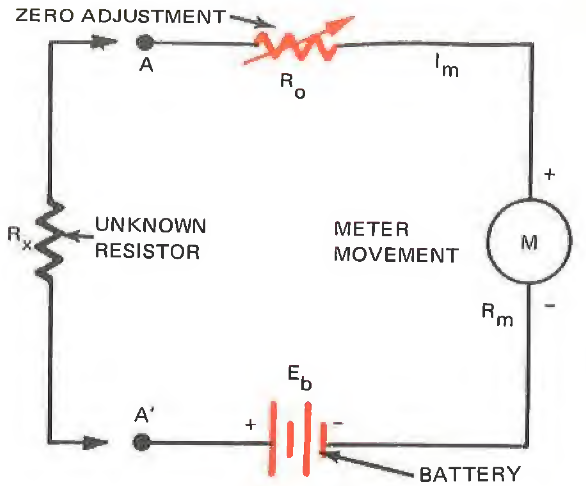


Fig. 12-5 A Series Ohmmeter

When an unknown value of resistance (R_x) is connected between points A and A′, the circuit current will be

$$I = \frac{E_b}{R_x + R_o + R_m}$$

If the instrument is to work properly, then the reading of the meter must be the same value as that of R_x. We may solve the above equation for this value by first multiplying each side by $(R_x + R_o + R_m)$:

$$IR_x + IR_o + IR_m - E_b$$

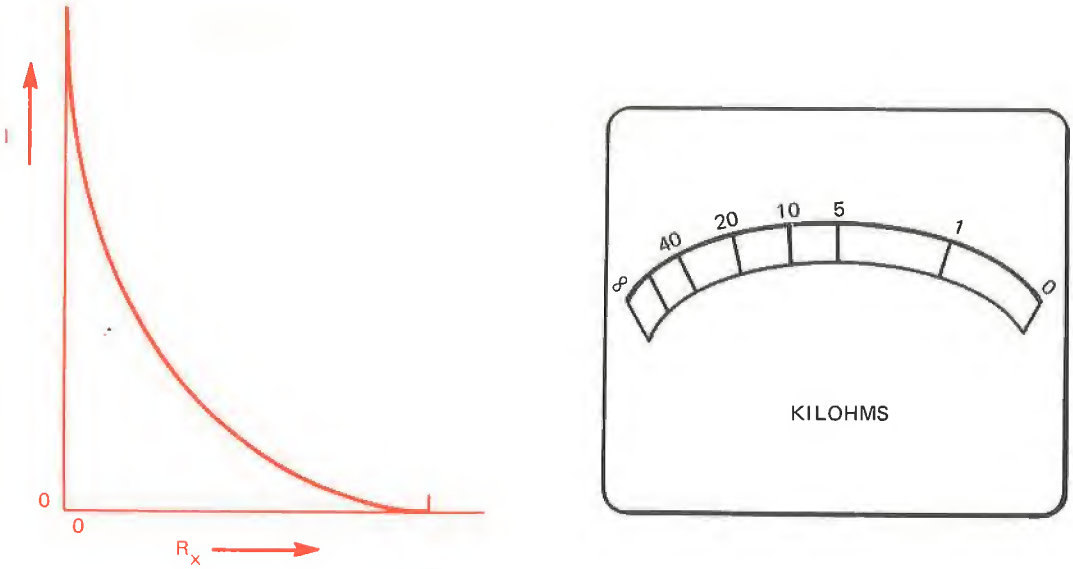
Then, subtracting $(IR_o + IR_m)$ from each side,

$$IR_x = E_b - I\,(R_o + R_m)$$

Finally, dividing both sides by I, we have

$$R_x = \frac{E_b}{I} - (R_o + R_m) \qquad \textbf{(12.6)}$$

From this equation we see that R_x (the



$$I = \frac{E_b}{R_o + R_m}$$

Fig. 12-6 Nonlinearity in an Ohmmeter Scale

meter reading) is not directly proportional to the current. The marking of the ohmmeter scale will therefore not be linear. Figure 12-6 shows a plot of meter current (I) versus meter reading (R_x) as well as a typical ohmmeter scale.

As in the case of other basic meters, an ohmmeter may have several ranges.

MATERIALS

1 0-1mA permanent-magnet moving-coil meter

1 Resistance decade box

1 Variable DC power supply

1 10kΩ-2W resistor

1 Multimeter

1 18-in. piece of No. 30 nichrome

1 12-in. steel rule

3 Sheets of linear graph paper

PROCEDURE

1. Examine the meter and identify the positive and negative terminals.
2. Check the mechanical zero adjustment of the meter and set the pointer for a zero reading.
3. Turn on the variable DC power supply and allow it to warm up; set the output for zero volts.
4. Connect the power supply, 10kΩ resistor and meter movement in series.
5. Carefully increase the power supply voltage until the meter reads 1.0 mA.

6. Connect the resistance decade box directly across the meter movement.
7. Adjust the resistance decade box until the meter reads 0.5 mA. Record the setting of the resistance decade box as R_m.
8. Disassemble the setup.
9. Connect the circuit shown in figure 12-7. Set the DC supply for zero output and use the resistance decade box for the 2kΩ resistor. Connect the nichrome wire shunt using about 6″ of No. 30 nichrome wire. DO NOT CUT OFF THE SURPLUS NICHROME WIRE

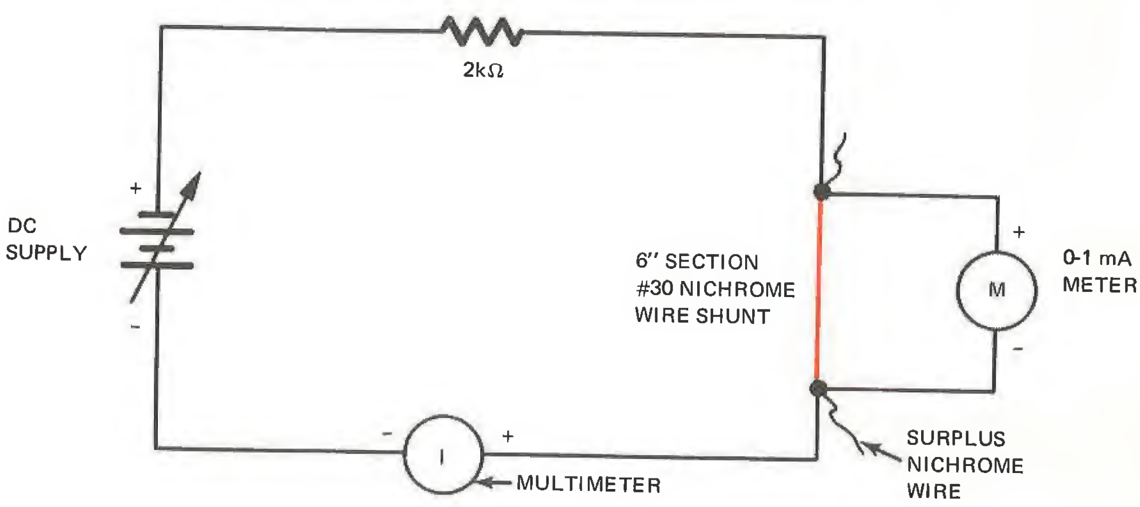


Fig. 12-7 The Experimental Ammeter

10. Slowly increase the voltage until the **multimeter** reads 10 mA. If the 0–1mA meter deflects off scale, check the connection of the shunt wire and shorten it if necessary. The 0–1 mA meter should not read full scale when 10 mA is flowing in the circuit.
11. Disconnect the 0–1 mA meter and lengthen the shunt wire a small amount. Reconnect the 0–1 mA meter and observe its reading. Repeat this process until the meter reads full scale. The multimeter should still read 10 mA.
12. By adjusting the DC supply, read and record the meter indication (I_m) for circuit currents (I_T) of 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0 and 10.0 mA.
13. Disassemble the setup. Carefully measure and record the length of the shunt wire.
14. Connect the circuit shown in figure 12-8. Use the resistance decade box for the multiplier resistor, R_V. The DC supply should be set for zero volts.
15. Set the DC supply to 10.0 volts as indicated by the multimeter. The 0–1 mA meter should not indicate full scale.
16. Reduce the setting of the resistance decade box until the 0–1 mA meter indicates full scale. The multimeter should still read 10.0 volts. Record the final value of the multiplier, R_V.

17. By varying the voltage, read and record the meter indication (E_m) for voltages (E_T) of 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0, 8.0, 9.0 and 10.0 volts.

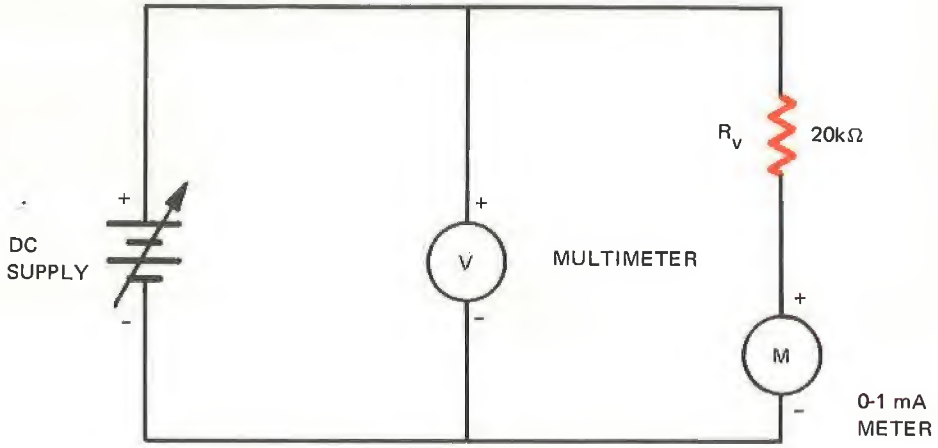


Fig. 12-8 The Experimental Voltmeter.

Ammeter Data

I_T (mA)	I_m
1.0	
2.0	
3.0	
4.0	
5.0	
6.0	
7.0	
8.0	
9.0	
10.0	

Voltmeter Data

E_T (volts)	I_m	E_m
1.0		
2.0		
3.0		
4.0		
5.0		
6.0		
7.0		
8.0		
9.0		
10.0		

Ohmmeter Data

R_x (ohms)	R_I
0	
100Ω	
500Ω	
1kΩ	
10kΩ	
20kΩ	
40kΩ	
40kΩ	
60kΩ	
100kΩ	

R_m (Meas)	R_s (Comp)	ℓ_s (Meas)	R_s (exp)	R_v (exp)	R_v (comp)	E_b (Meas)	E_b (Comp)

Fig. 12-10 The Data Table

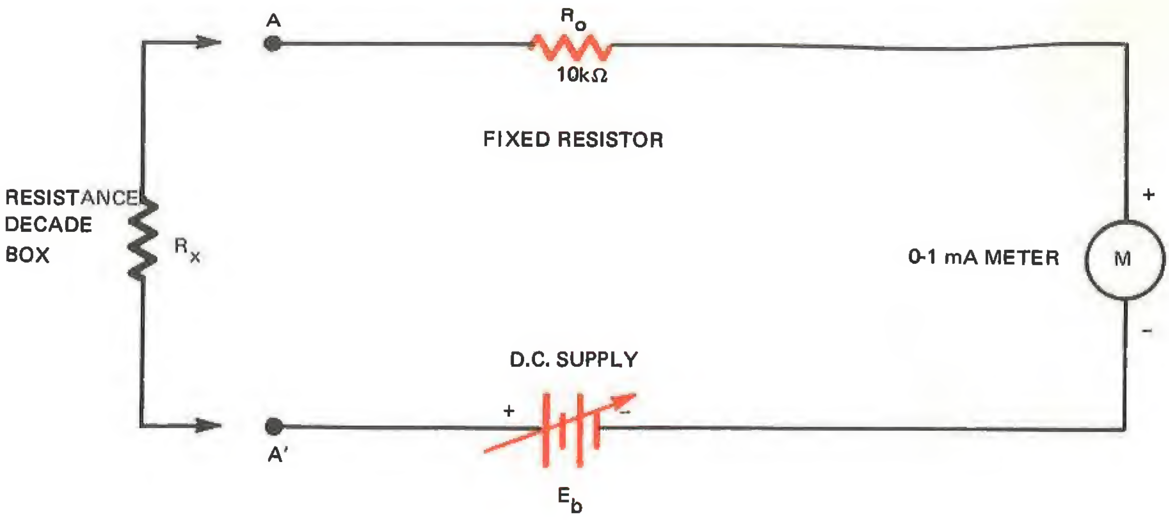


Fig. 12-9 The Experimental Ohmmeter

18. Disassemble the voltmeter circuit and set up the circuit shown in figure 12-9.
19. Short circuit the points A and A′ and adjust the DC supply for a full scale reading on the meter. Measure and record the voltage, E_b.
20. Remove the short from points A and A′ and connect the resistance decade box. Read the percent of maximum deflection of the meter and record this value as $R_{\%}$ for values of R_x of 100Ω, 500Ω, 1kΩ, 5kΩ, 10kΩ, 20kΩ, 40kΩ, 60kΩ, and 100kΩ.
21. Using the measured value of R_m and the meter full scale current, compute and record the value of R_s (comp) required for a 10 mA range.
22. Compute and record the value of the resistance of the nichrome wire shunt R_s (exp) using the measured length of the wire and a wire table.
23. With the measured value of R_m and the meter full scale current, compute and record the value of a multiplier resistor, R_v (comp) necessary for a 10 volt range.
24. Use equation 12.5 to determine the voltage necessary to construct an ohmmeter with R_o value of 10k. Record this value of E_b (comp).
25. On three separate sheets of linear graph paper, plot I_T versus I_m, E_T versus E_m, and R_x versus R_I.

ANALYSIS GUIDE. In analyzing these data, you should consider two main points. These points are:

a. How well do the various computed values compare with the corresponding measured values?
b. Are the curves plotted consistent with appropriate mathematical analysis?

PROBLEMS

1. Using a 1 mA meter with an internal resistance of 100 ohms, a 10.1-volt battery and an assortment of resistors, draw a neat circuit diagram of a single instrument with D C ranges of 0–1 mA, 0–10 mA, 0–10 volts, and 0–100kΩ. (Show all values.)

2. Make a sketch of a meter scale that could be used with the instrument in problem one.

3. What was the sensitivity of the voltmeter circuit used in the experiment?

4. Explain in your own words why an ohmmeter scale is nonlinear.

INTRODUCTION. In general electrical laboratory work, voltmeters and ammeters play a central role. In some cases the technique employed in making measurements does have an effect on the results. In this experiment we shall examine this aspect of laboratory work.

DISCUSSION. Let us suppose that we have a simple series circuit like the one shown in figure 13-1. Further suppose that we wish to measure the voltage across, and the current through the resistor R_2. The two parameters which we wish to measure will be equal to

$$I = \frac{E_T}{R_1 + R_2} \qquad (13.1a)$$

and

$$E_2 = E_T \frac{R_2}{R_1 + R_2} \qquad (13.1b)$$

When we undertake to actually measure these two quantities, we are faced with a choice. We may decide to make the measurements separately using the circuits shown in figure 13-2 or we may elect to make both measurements simultaneously employing either figure 13-3(a) or 13-3(b). If the separate method of figure 13-2 is used, the results will be

$$I = \frac{E_T}{R_1 + R_2 + R_I} \qquad (13.2a)$$

and

$$E_2 = E_T \frac{R_2 R_v}{R_1 R_v + R_1 R_2 + R_2 R_v} \qquad (13.2b)$$

where R_I and R_v are the resistances of the current meter and the voltmeter respectively.

Equation 13.2a would be exactly equal to 13.1a only if R_I were zero ohms. On the other hand, we can compare equations 13.1b

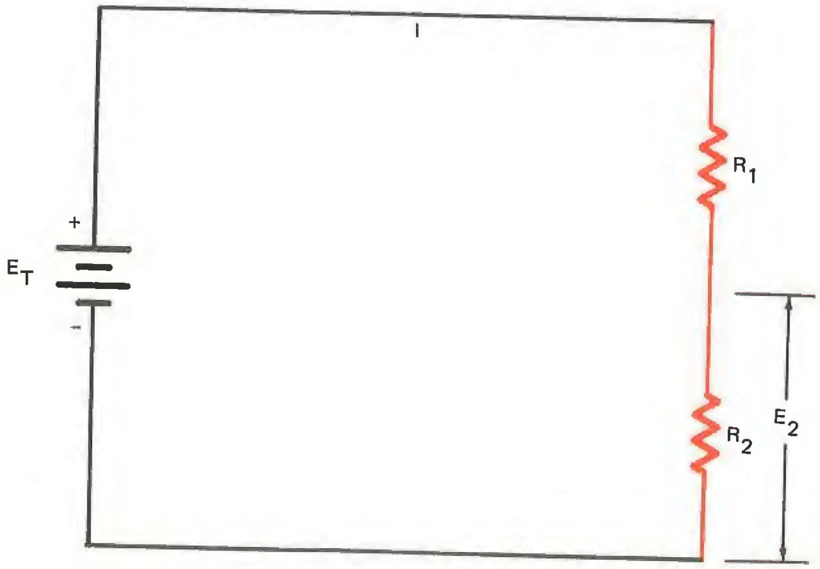
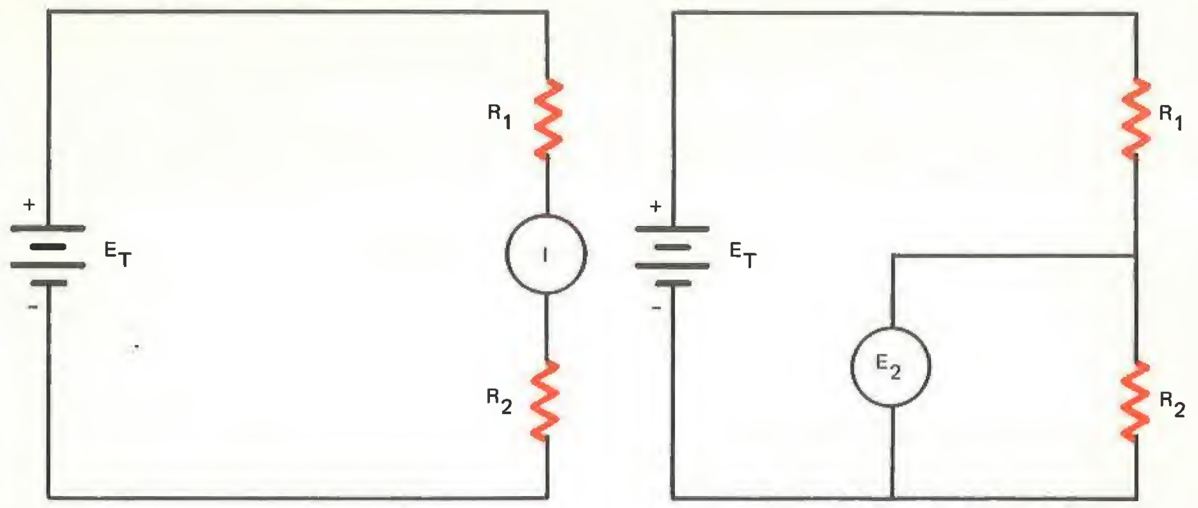


Fig. 13-1 A Simple Series Circuit

 ELECTRONICS/ELECTRICITY



(a) Current Measurement *(b) Voltage Measurement*

Fig. 13-2 Measuring the Quantities Separately

and 13.2b most favorably if R_V is very large indeed. If R_V is much larger than either R_1 or R_2, then the denominator of 13.2b can be seen to approach $R_1R_V + R_2R_V$. That is,

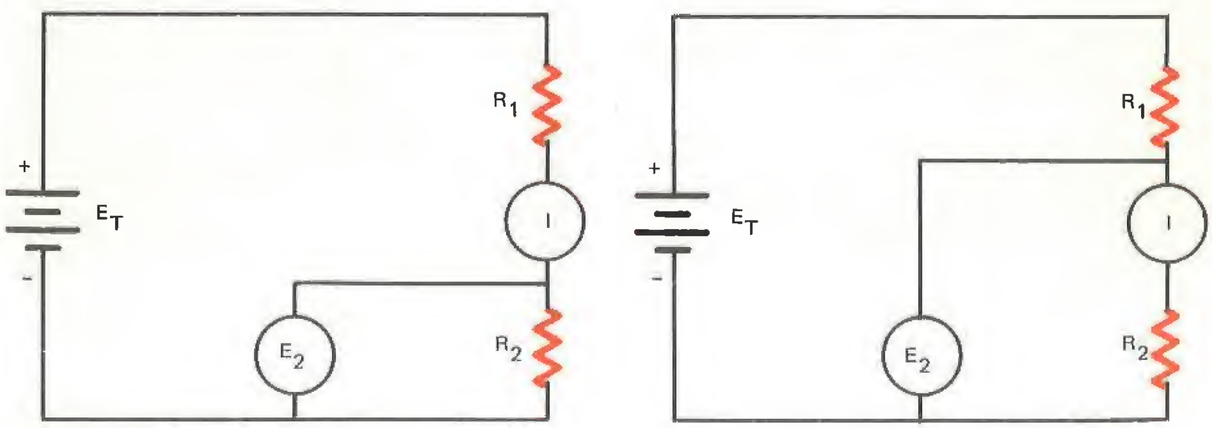
$$R_1R_V + R_1R_2 + R_2R_V \approx R_1R_V + R_2R_V$$

$$\text{if,} \quad R_V \gg R_1 \text{ and } R_2$$

When this is the case, R_V cancels in the numerator and denominator and eq. 13.2b becomes the same as 13.1b.

We see then that the conditions for accuracy when using the separate method of measuring I and E_2 are that the current meter should have the lowest possible resistance, while the voltmeter should have the highest possible resistance.

If we choose to make both measurements simultaneously, we must choose one of the circuits shown in figure 13-3.



(a) Short Shunt *(b) Long Shunt*

Fig. 13-3 Alternative Simultaneous Meter Connections

No Meters		Separate Meas.		Short Shunt		Long Shunt	
E_2	I	E_2	I	E_2	I	E_2	I
3 volts	10 mA	2.48 volts	9.1 mA	2.24 volts	9.68 mA	2.9 volts	7.24 mA

Fig. 13-4 Data From the Example

Let us examine each alternative in turn. The short shunt method of figure 13-3a will give us results of

$$I = \frac{E_T}{R_1 + R_I + \dfrac{R_2 R_v}{R_2 + R_v}} \qquad (13.3a)$$

and

$$E_2 = E_T \frac{R_2 R_v}{R_1 R_2 + R_1 R_v + R_I R_2 + R_I R_v} \qquad (13.3b)$$

Inspection of these equations reveals that these results would differ both from the original circuit and from the results of the separate measurement method. However, if $R_I = 0$ and $R_v = \infty$, then these results become the same as the original conditions.

This conclusion can be seen by removing all terms involving R_I (they will be zero), then allowing R_v to be so large that $R_v + R_2$ in Eq. 13.3a will be approximately R_v only. R_v then cancels in Eq. 13.3a and 13.3b becomes equal to 13.2b, which may be simplified as before.

In the case of the long shunt circuit of figure 13-3b, the current and voltage readings will be

$$E_2 = E_T \frac{R_I R_v + R_2 R_v}{R_1 R_2 + R_1 R_I + R_1 R_v + R_I R_v + R_2 R_v} \qquad (13.4b)$$

and

$$I = \frac{E_2}{R_I + R_2}$$

$$= E_T \frac{R_v}{R_1 R_2 + R_1 R_I + R_1 R_v + R_I R_v + R_2 R_v} \qquad (13.4a)$$

Once again we observe that these results are not equal to the ones found before. However, in this case too, if $R_I = 0$ and $R_v = \infty$, then the results become equal to the original case.

Perhaps this rather confusing state of affairs will be clarified by tabulating a numerical example. Let us suppose that $E_T = 10.0$ volts, $R_1 = 700$ ohms and $R_2 = 300$ ohms. Moreover, suppose that $R_I = 100$ ohms and $R_v = 1000$ ohms. The results of the four cases would then be as given in figure 13-4. If $R_I = 0$ and $R_v = \infty$ are chosen, then each of the four sets of results are equal to the "No Meters" condition given in figure 13-4.

From the discussion and example above, it should be apparent that the choice of meter characteristics and technique can have considerable effect on measured results.

MATERIALS

2 Multimeters (VOM)

1 0-1 mA permanent-magnet, moving-coil meter

1 Variable DC power supply

1 Resistance decade box

1 10kΩ 2W resistor

1 5kΩ 2W resistor

PROCEDURE

1. Compute the value of a multiplier resistor to be used with the 0-1 mA meter in a 0-10 volt voltmeter circuit. Record this value as R_V in the data table, figure 13-5a.
2. Construct the voltmeter using the resistance decade box as the multiplier.
3. Assemble the circuit shown in figure 13-4.

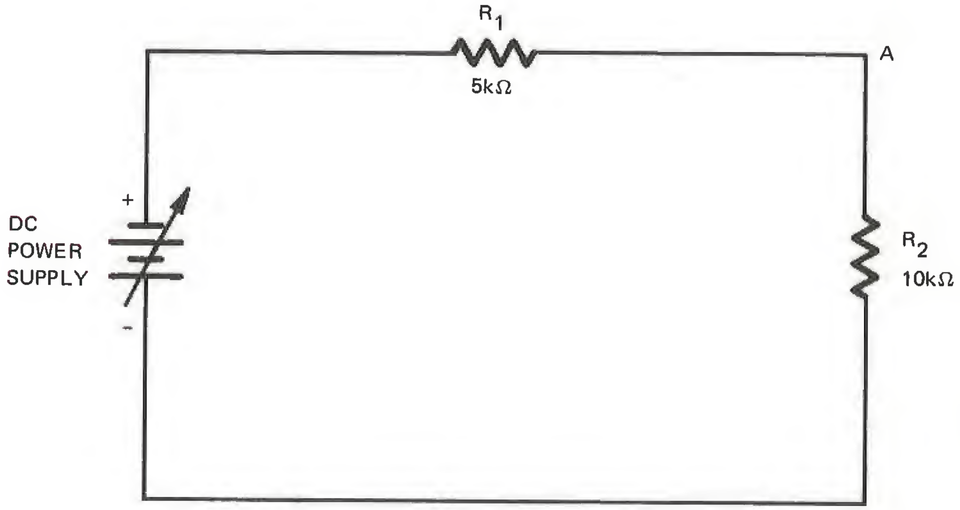


Fig. 13-4 The Experimental Circuit

4. Set the DC power supply for an output of about 12 volts. Record this quantity as E_T in the data table. (Fig. 13-5a)
5. Set up the multimeter for measuring DC current and insert into the circuit at point A in figure 13-4.
6. Measure and record the circuit current as I_T in the data table, figure 13-5b.
7. Connect the voltmeter, constructed in step 2, across the 10k resistor only and record the readings of both the current meter (I_2) and the voltmeter (E_2) in figure 13-5b.
8. Reconnect the voltmeter across both the current meter and the 10k resistor. Record the readings as E_2' and I_2'.
9. Move the voltmeter to the position required to measure the voltage across the 5kΩ resistor only. Record the readings as I_1 and E_1 in the data table.

R_V Comp.	E_T Meas	R_1 Meas	R_2 Meas	I_T Comp	E_1 Comp	E_2 Comp	S_1 Comp

Fig. 13-5a Circuit Data

Qty.	Multimeter I & Const. V meter	Multimeter I & VOM V	0-1 mA I & Multimeter V
I_T			
I_1			
E_1			
I_1'			
E_1'			
E_1''			
I_2			
E_2			
I_2'			
E_2'			
E_2''			

Fig. 13-5b Instrument Comparison Data

10. Again move the voltmeter to indicate the voltage across both the 5kΩ resistor and the current meter. Record the meter readings as I_1' and E_1' in the data table.

11. Remove the current meter from the circuit and measure the voltage across the 5kΩ re-resistor and the 10kΩ resistor individually. Record these values as E_1'' and E_2'', respectively.

12. Repeat steps 5 through 11 using the VOM to measure the voltages.

13. Disconnect the voltmeter circuit constructed in step 2 and insert the 0-1 mA meter into the circuit at point A of figure 13-4.

14. Repeat steps 6 through 11 using the multimeter for the voltage measurements and the 0-1 mA meter for the current readings.

15. Disassemble the test circuit and measure and record the ohmic values of R_1 and R_2. Record the values in figure 13-5a.

16. Calculate and record (in figure 13-5a), using only the applied voltage and the measured resistance values, the circuit current and the voltage drops across each of the resistors.

17. Determine and record the sensitivities of the constructed voltmeter. Record this value as S_i in the data table, figure 13-5a.

ANALYSIS GUIDE. In analyzing these data, you should consider the following points:

a. To what extent did the measurement techniques used affect the results?
b. Does instrument sensitivity have an effect on measurement accuracy?
c. Was one *technique* superior to the others? Why?
d. Is the sum of the two readings taken in step 11 equal to the applied voltage? Explain why.

PROBLEMS

1. Assume that the resistances in figure 13-5 were 50 and 100 ohms instead of 5kΩ and 10kΩ. If the 0-1 mA current meter and the multimeter (voltmeter) were connected in long shunt across the 100 ohm resistor, what should the readings be?
2. What would be the results in problem 1 if the meters were connected short shunt?

experiment 14 ELECTROSTATIC CHARGES

INTRODUCTION. *Electrostatic charge* was perhaps the first observed electrical phenomenon and still holds a position of vast importance in electricity. In this experiment we shall examine the manner in which charges are stored and distributed in capacitive circuits.

DISCUSSION. The basic unit of electric charge is the *electron*. However, since the amount of charge on a single electron is very small, we usually deal with a larger unit called a *coulomb*. One coulomb is equivalent to the charge of 6.24×10^{18} electrons.

We can define electric current (I) as being the number of coulombs of charge passing a given point per unit of time: that is,

$$I = \frac{Q}{t} \qquad (14.1)$$

where I is the current in amperes, Q is the amount of charge in coulombs, and t is time in seconds.

The amount of material that a container can hold is called the *capacity* of the container. For instance, we might say that a cylindrical water tank 20 ft. high has a capacity of 10,000 gallons. The *capacitance* of the tank is the quantity of water required to raise the level one unit. For this tank we could say that the capacitance was

$$C = \frac{\text{capacity}}{\text{height}} = \frac{10{,}000 \text{ gal}}{20 \text{ ft}} = 500 \text{ gal/ft.}$$

Alternately, we could express the capacitance as the ratio of the capacity to the *pressure head* since the pressure head is directly related to the level. This is what we, in fact, do in electricity. We can, therefore, define electrical capacitance as the ratio of the quantity of charge (in coulombs) to the pressure (in volts). That is

$$C = \frac{Q}{E} \qquad (14.2)$$

Electrical capacitance exists whenever two conductors are separated by a *dielectric material* (*insulator*). The value of capacitance in farads is proportional to the area of the conductors and inversely proportional to

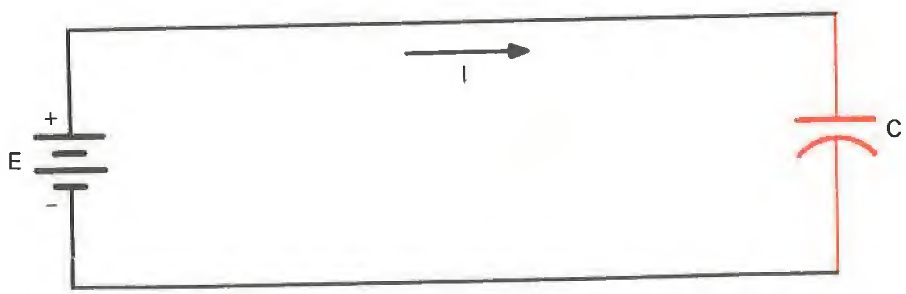


Fig. 14-1 Charging a Capacitor

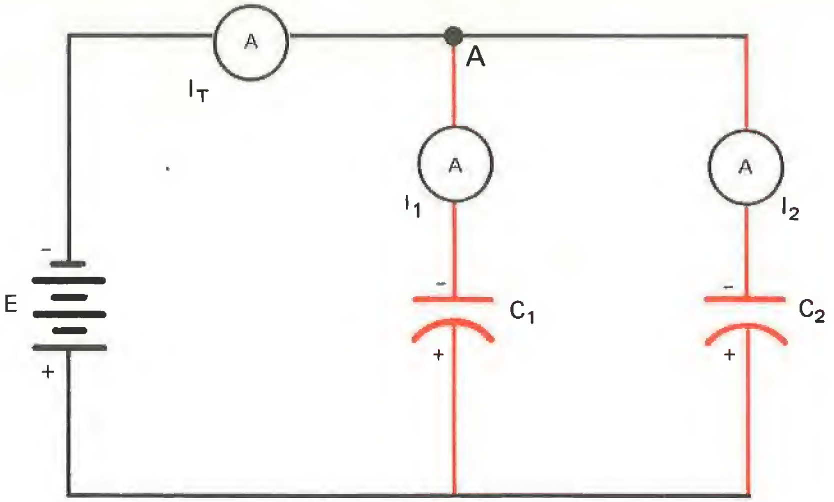


Fig. 14-2 Capacitors in Parallel

the distance between them. That is,

$$C = \epsilon \frac{A}{d}$$

where the constant of proportionality, ϵ, is called the *permitivity* of the dielectric.

If we connect a capacitor across a source, as shown in figure 14-1, current will flow momentarily into the capacitor, making the voltage across it equal to the source voltage. The charge stored by the capacitor can be found using equation 14.2.

$$\boxed{Q = CE}$$

Q is in coulombs, C is in farads, and E is in volts.

If we place two (or more) capacitors in parallel across a source, as in figure 14-2, the currents flowing at junction A can be seen to be related by

$$I_T = I_1 + I_2$$

If we multiply each side of this equation by t, the result will be

$$I_T t = I_1 t + I_2 t$$

However, we see from equation 14.1 that

$$It = Q$$

Therefore,

$$\boxed{Q_T = Q_1 + Q_2} \qquad \textbf{(14.3)}$$

must account for the charge distribution in a parallel circuit. In other words, the total charge delivered by the source is equal to the sum of charges on the individual capacitors.

From equation 14.2, we observe that

$$Q_T = C_T E,\ Q_1 = C_1 E \text{ and } Q_2 = C_2 E.$$

Substituting the CE products into equation 14.3 gives us

$$C_T E = C_1 E + C_2 E.$$

Dividing each side by E renders

$$\boxed{C_T = C_1 + C_2} \qquad \textbf{(14.4)}$$

which, of course, means that the total capacitance of several capacitors in parallel is equal to the sum of the individual capacitances.

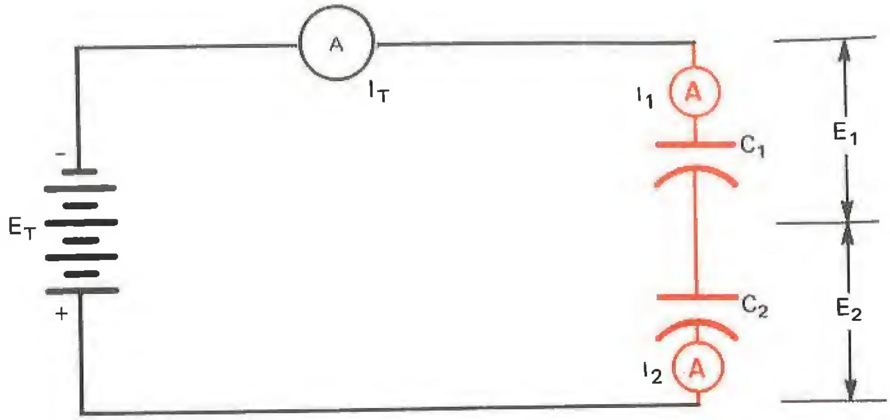


Fig. 14-3 Capacitors in Series

Alternately, we can connect capacitors in series across a source as indicated in figure 14-3. Since the circuit is a simple series connection, the current through each element is identical:

That is,

$$I_t = I_1 = I_2.$$

And if we multiply these quantities by t, we have

$$I_T t = I_1 t = I_2 t$$

or in other words,

$$\boxed{Q_T = Q_1 = Q_2} \qquad (14.5)$$

which tells us that the charge on each capacitor and the total charge drawn from the source are all equal in value.

Returning to figure 14-3, we may observe that

$$E_T = E_1 + E_2.$$

But, from equation 14.2,

$$E_T = \frac{Q_T}{C_T},\ E_1 = \frac{Q_1}{C_1} \text{ and } E_2 = \frac{Q_2}{C_2}.$$

Substituting these relationships into the one involving voltage yields

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}.$$

However, we have determined (Eq. 14.5) that the charges are all equal in this case. They may, therefore, be canceled in each term, providing

$$\boxed{\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}} \qquad (14.6)$$

This equation informs us that in a series connection, the reciprocal of the total capacitance is equal to the sum of the reciprocals of the individual capacitances.

While equation 14.6 holds for any number of series capacitors, it is unusual to have more than two in series at one time. Therefore, we may rearrange equation 14.6 by taking the common denominator,

$$\frac{1}{C_T} = \frac{C_1 + C_2}{C_1 C_2}$$

or

$$\boxed{C_T = \frac{C_1 C_2}{C_1 + C_2}} \qquad (14.7)$$

which is, of course, the familiar *product over sum* equation.

MATERIALS

1 Wrist watch with a sweep-type second hand (supplied by the student)

1 Variable DC power supply

2 10 μF 600V oil filled capacitors

1 Single-pole, single-throw switch

1 Multimeter (internal resistance 10 megohms or above)

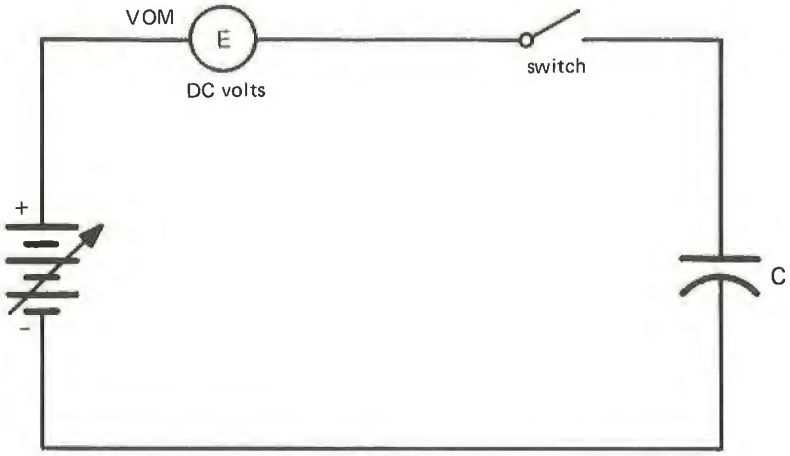


Fig. 14-4. The Experimental Circuit

PROCEDURE

1. Connect the circuit shown in figure 14-4 using a single capacitor.
2. Short circuit the capacitor with a clip lead.
3. Set the VOM to the 15-volt DC range and close the switch.
4. Adjust the source voltage for reading of 10.0 volts on the VOM.
5. Determine the input resistance of the VOM. Compute the current that must flow through the meter to give the reading set in step 4. Record this value as I in part 1.

NOTE: The following steps (6 through 9) will require some skill before they can be carried out smoothly. It is, therefore, suggested that you practice several times before attempting to do them for data. In practicing the operation, always start with the capacitor short-circuited and the VOM reading 10.0 volts.

6. Choose some convenient starting point on the seconds scale of your wrist watch.
7. At the chosen time, quickly remove the short circuit from the capacitor.
8. As the capacitor charges, the charging current (and therefore the VOM reading) will tend to decrease. To offset this effect, you will need to gradually increase the output of the DC source to maintain the VOM reading at 10 volts. **It is very important that this be done smoothly.**

9. After the capacitor has charged for 90 seconds (1 minute, 30 seconds), open the switch in the experimental circuit.

10. Record the charging time (T) in part 1 of the data table.

11. Using the charging current and time, compute the total charge on the capacitor(s) and record it in the data table.

12. Remove the VOM and the capacitor(s) from the circuit. **Be careful not to touch the capacitor terminals as that would allow some of the charge to leak off through your body.**

13. Quickly measure the voltage stored by the capacitor. Do not leave the VTVM connected to the capacitor longer than necessary to get the reading. Record this value as E in the data table.

14. Using the values of Q and E, compute the capacitance of the capacitor and record it in the data table.

15. Repeat steps 1 through 14 using the other capacitor and record the results in part 2 of the data table.

16. Repeat steps 1 through 12 using the two capacitors in parallel. Record the data in part 3 of the table.

17. Measure and record the voltage across each capacitor, (E_1 and E_2).

18. Using C_1 and C_2 from parts 1 and 2 respectively, compute the charge on each capacitor (Q_1 and Q_2) and record it in the data table.

19. Using the appropriate equation from the discussion (14.3, 14.5), compute the value of the total charge (Q'_T).

20. Compute the percent difference between Q_T and Q'_T.

21. Using the value Q_T and the appropriate voltage, compute C_T.

22. Using the values of C_1 and C_2 found in parts 1 and 2, compute C'_T with the appropriate equation from the discussion (14.4, 14.6, or 14.7).

23. Compute the percent difference between C_T and C'_T.

24. Repeat steps 1 through 12 and 17 through 23 using the two capacitors in series. Record this data in part 4 of the table.

ANALYSIS GUIDE. In analyzing this data, pay particular attention to the validity of the equations for C_T and Q given in the discussion. Also, discuss the method used to measure the charging current.

I	T	Q_T	E	C_1

Part 1

I	T	Q_T	E	C_2

Part 2

I	T	Q_T	E_1	E_2	Q_1	Q_2	Q'_T	% Diff Q_T	C_T	C'_T	% Diff C_T

Part 3

I	T	Q_T	E_1	E_2	Q_1	Q_2	Q'_T	% Diff Q_T	C_T	C'_T	% Diff C_T

Part 4

Fig. 14-5 The Data Table

PROBLEMS

1. Consider two series capacitors charged from a source such that

$$Q_T = Q_1 = Q_2$$

If we carefully disconnect the capacitors and reassemble them in parallel, their charges (which are equal) become additive. That is

$$Q_1 + Q_2 = 2Q_T$$

It would seem then that by reconnecting them this way we can retrieve twice as much charge as was supplied by the source.

Write a paragraph explaining how this could possibly be true.

2. A 20 μF capacitor is charged to 100 volts. It is then removed from the source and connected across a 10 μF capacitor which has been similarly charged to 200 volts. What will be the voltage across each capacitor after the charges redistribute?

experiment 15 CAPACITOR CHARGING

INTRODUCTION. When a capacitor is charged through a resistance, a predictable amount of time is required to transfer a given charge. In this experiment we shall examine the relationship between charge and time in an RC circuit.

DISCUSSION. Consider the simple RC circuit of figure 15-1.

At the instant when the switch S is closed, the voltage across the capacitor is zero. Since Kirchhoff's law must be satisfied even at that instant, the full source voltage must be impressed across the resistor. The *initial* current in the circuit must, therefore, be

$$I_o = \frac{E_o}{R} \qquad (15.1)$$

As the capacitor begins to charge, the voltage across the resistor tends to decrease. In other words, the current in the circuit decreases as the capacitor charges.

If we prevent the current from changing by varying the voltage, *it is possible to charge the capacitor at a constant rate.*

$$Q_c = I_o\tau$$

Substituting E_o/R into this relationship for I_o renders

$$Q_c = \frac{E_o}{R}\tau \qquad (15.2)$$

As the capacitor charges, the value of the charge increases according to

$$Q_c = CE$$

and must eventually reach the value,

$$Q_c = CE_o$$

At this point we can substitute CE_o into equation 15.2 for Q_c and get

$$CE_o = \frac{E_o}{R}\tau$$

Canceling E_o and solving for τ renders

$$\boxed{\tau = RC} \qquad (15.3)$$

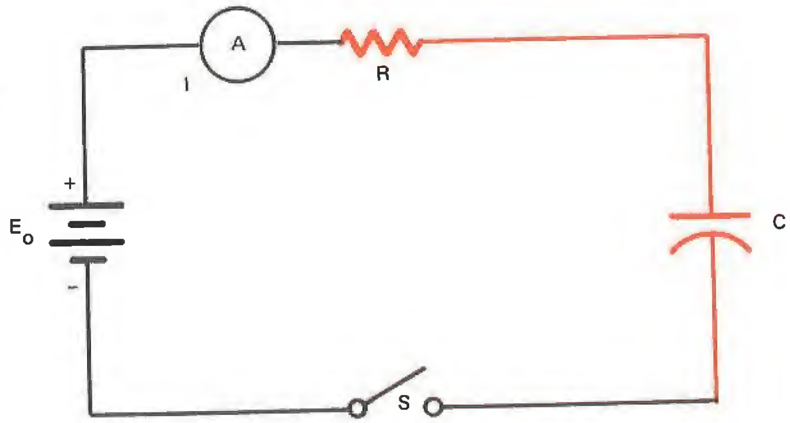


Fig. 15-1 An RC Charging Circuit

In other words, the time required to charge the capacitor *at a constant rate* to a voltage equal to E_o can be found by taking the product of R times C. Time, τ, will be in seconds if R is in ohms and C is in farads.

The length of time (τ = RC) discussed above is called one *time constant* for the RC circuit.

As mentioned above, if no attempt is made to hold the charging current constant, it will decrease as the capacitor charges. Figure 15-2 shows a plot of current versus time for the condition where E_o and R are constant.

The equation describing such a relationship is

$$I = I_o \epsilon^{-\tau/RC} = \frac{E_o}{RC} \epsilon^{-\tau/RC} \qquad (15.4)$$

where ϵ is the natural number 2.718.

With the circuit current given in equation 15.4, the voltage drop across the resistor will be

$$\boxed{E_R = IR = E_o \epsilon^{-\tau/RC}} \qquad (15.5)$$

By Kirchhoff's law, the voltage across the capacitor is

$$E_c = E_o - E_R$$

Substituting the value for E_R into this relationship gives

$$E_c = E_o - E_o \epsilon^{-\tau/RC}$$

or

$$\boxed{E_c = E_o (1 - \epsilon^{-\tau/RC})} \qquad (15.6a)$$

If the capacitor has been charged to an initial voltage E_I before the switch in figure 15-1 is closed at τ = 0, then the equation describing the capacitor voltage becomes

$$E_c = E_o (1 - \epsilon^{-\tau/RC}) + E_I \epsilon^{-\tau/RC}$$

or

$$E_c = \epsilon_I + (E_o - E_I)(1 - \epsilon^{-\tau/RC}) \qquad (15.6b)$$

Using this equation, the value of the voltage across the capacitor may be found for any time τ.

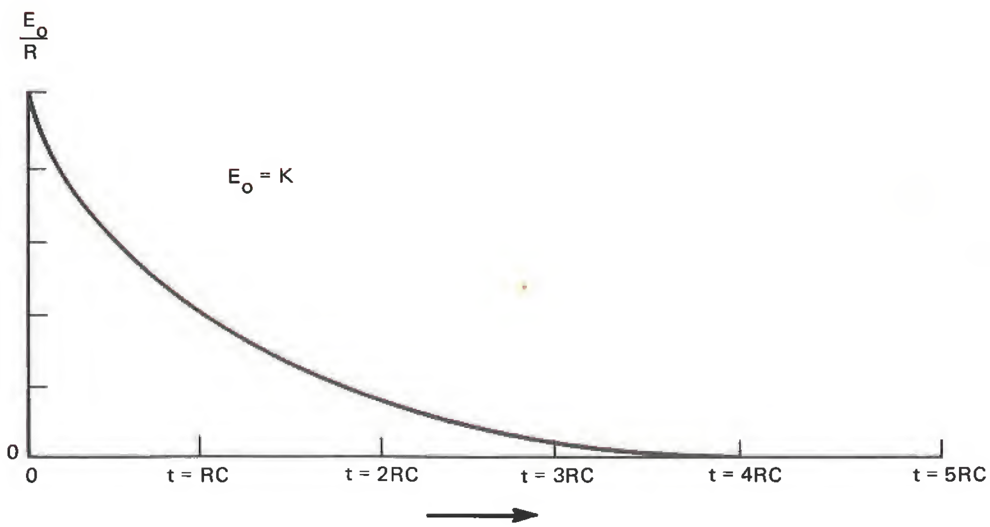


Fig. 15-2 Current in an RC Circuit

If we take a charged capacitor and connect it in a circuit of the type shown in figure 15-3, the capacitor will discharge through the resistance when the switch is closed.

In such a case, the initial current will be

$$I_o = \frac{E_o}{R}$$

and the subsequent current will decrease according to

$$I = I_o\,\epsilon^{-\tau/RC} = \frac{E_o}{R}\,\epsilon^{-\tau/RC}$$

The voltage across the resistor will vary according to

$$E_R = IR = E_o\,\epsilon^{-\tau/RC}$$

And since the resistor and capacitor are in parallel, the capacitor voltage will also change according to the relationship:

$$E_c = E_o\,\epsilon^{-\tau/RC} \qquad \textbf{(15.7)}$$

Using this equation, we may determine the voltage across the capacitor any time, τ, as it discharges.

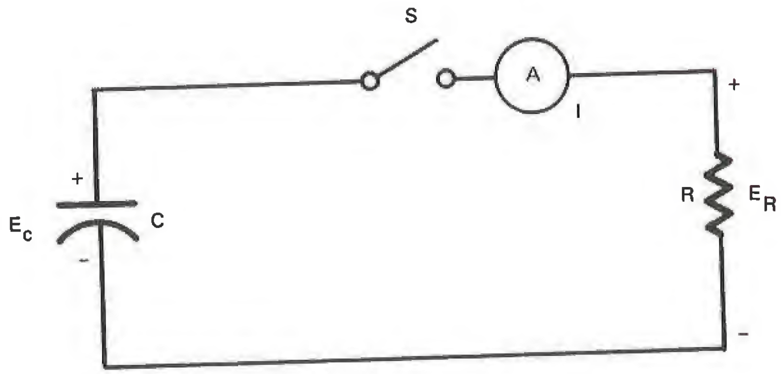


Fig. 15-3 An RC Discharge Circuit

MATERIALS

- 1 Variable DC power supply
- 1 Multimeter (11 megohm resistance or above)
- 1 10 μF oil-filled capacitor
- 1 Switch (SPST)
- 1 Watch with sweep second hand (supplied by student)
- 2 Sheets of linear graph paper

PROCEDURE

1. Connect the circuit shown in figure 15-4.
2. Compute the value of one time constant and enter it in the data table (Part 1).
3. Short circuit the capacitor with a clip lead, close switch S, and adjust the power supply for a VOM reading of 12.0 volts.

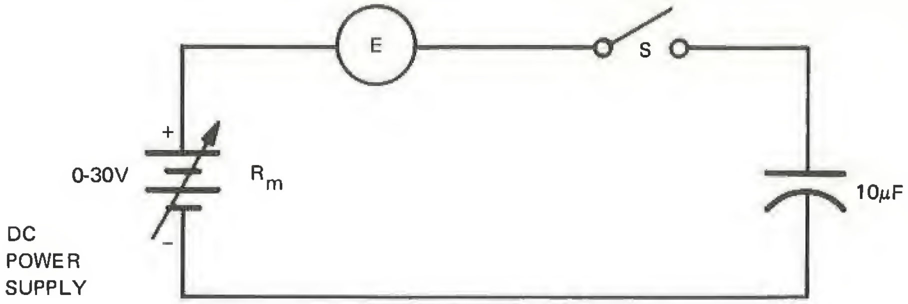


Fig. 15-4 The Experimental Circuit

4. Quickly remove the short circuit from the capacitor and allow to charge for one time constant. Keep the VOM reading constant at 12.0 volts by continuously adjusting the source voltage. At the end of one time constant, open switch S.

5. Using the VOM, measure and record the value of the capacitor voltage (E_c) in Part 1 of the data table. Make this measurement quickly and do not leave the meter connected across the capacitor longer than necessary.

6. Determine the meter input resistance and compute the value of the charging current. Record it as I_o in the data table (Part 1).

7. Using I_o and the charging time, compute and record the value of the charge Q on the capacitor.

8. Using Q and E_c, compute the capacitance of the capacitor. Record the result in the data table.

9. Compute and record the percent difference between E_o and E_c.

10. Reconnect the experimental circuit as before (figure 15-4) and re-establish the conditions of step 3.

11. Remove the short circuit from the capacitor and record the VOM reading. Do not change the value of the power supply output this time. Starting at T = 0, record the VOM reading every 30 seconds for 10 minutes. Record these values in part 2 of the data table.

12. Compute the value of the circuit current for each VOM reading taken in step 12.

13. Using Kirchhoff's law, $E_c = E_o - E_R$, compute the capacitor voltage for each VOM reading taken in step 12.

14. On a single sheet of graph paper, plot the values of current and capacitor voltage versus time.

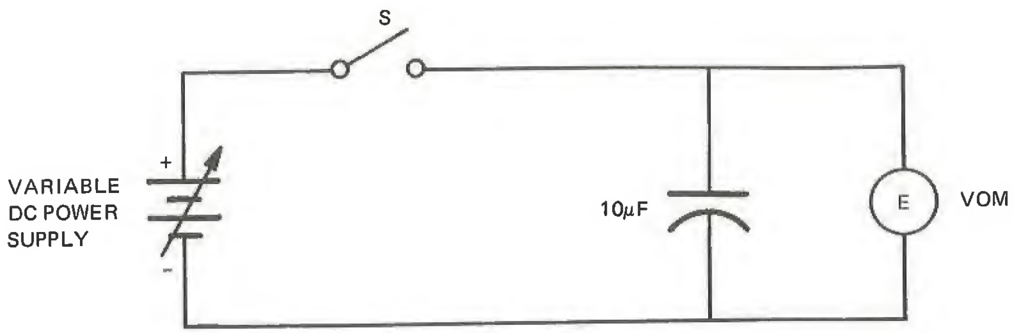


Fig. 15-5 The Second Experimental Circuit

15. Assemble the circuit shown in figure 15-5.
16. With the switch closed, set the power supply for a VOM reading of 12.0 volts. Record this value in the data table, part 3, opposite T = 0.
17. Open the switch and record the VOM reading every 30 seconds for 10 minutes.
18. On a second sheet of graph paper, plot the values of the capacitor voltage taken in step 17 versus time.
19. Using the measured values of E_o, C and the meter input resistance for R, plot the equation

$$E_c = E_o\,(1 - \epsilon^{-\tau/RC})$$

on the sheet of graph paper used in step 14.

20. Similarly, plot $E_c = E_o\,\epsilon^{-\tau/RC}$ on the sheet of graph paper used in step 18.

ANALYSIS GUIDE. In the analysis of these data you should discuss the extent to which the plots of the equations (steps 20 and 21) agreed with the plots of the measured data.

PROBLEMS

1. How many time constants are required to charge a capacitor to 95% of the supply voltage?
2. Is your answer in problem one true for any values of R, C, and E_o?
3. Draw a sketch of the voltage across the capacitor shown in figure 15-6 if the switch changes position every second.

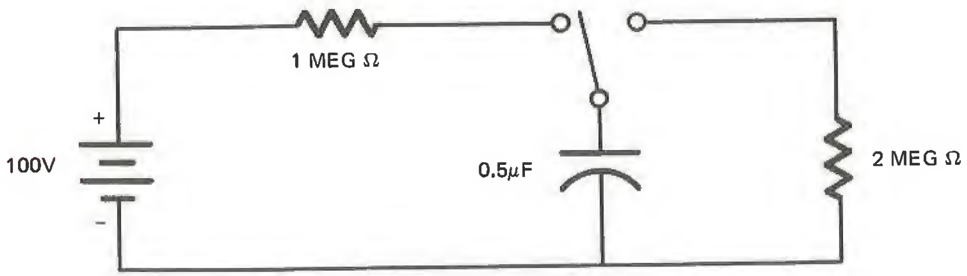


Fig. 15-6 Circuit for Problem 3

RC	E_o	E_c	I_o	Q	C	% Diff E
	12.0V					

Part 1

T (Min)	E_R (VOM)	I	E_c
0			
0.5			
1.0			
1.5			
2.0			
2.5			
3.0			
3.5			
4.0			
4.5			
5.0			
5.5			
6.0			
6.5			
7.0			
7.5			
8.0			
8.5			
9.0			
9.5			
10.0			

Part 3

T (Min)	E_c
0	
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	
5.5	
6.0	
6.5	
7.0	
7.5	
8.0	
8.5	
9.0	
9.5	
10.0	

Part 2

Fig. 15-7 The Data Table

experiment **16** *OSCILLOSCOPE MEASUREMENTS*

INTRODUCTION. The *cathode ray oscilloscope* is used as an instrument of electrical measurement about as much as basic meters. In this experiment we shall examine the operational techniques appropriate for use with this instrument.

DISCUSSION. The essential device employed in a cathode ray oscilloscope (CRO) is the *cathode ray tube* (CRT). A pictorial sketch of such a tube is shown in figure 16-1.

The operation of the CRT can be explained by considering three basic sections:

1. The *electron gun* generates a narrow beam of electrons which is directed along the length of the tube.
2. If we apply a voltage across the *horizontal deflection plates* in the deflection section, the beam will be bent away from the negative plate and toward the positive plate. This tends to cause the beam to move back and forth horizontally across the tube. Similarly, the *vertical deflection plates* can bend the beam up and down depending on the polarity and magnitude of the voltage applied to them.
3. When the electron beam strikes the *fluorescent screen,* a spot of light is generated indicating the location of the beam. The overall effect is that the light spot on the screen moves on the screen in a manner dependent upon the voltages applied to the horizontal and vertical deflection plates.

If the extent of the deflection per volt is known for a particular CRT, then the distance that the spot moves on the screen can be used to measure voltage.

The operation of a practical oscilloscope can be better understood by considering the functional block diagram shown in figure 16-2.

The *vertical deflection control circuits* (on the lower left of figure 16-2) allow us to position the spot vertically and to calibrate the vertical deflection.

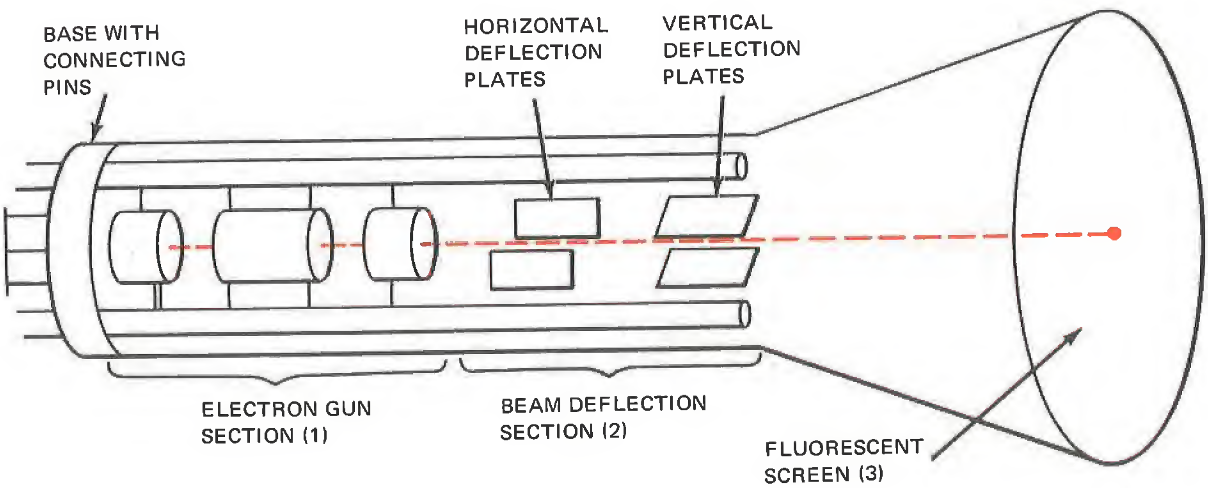


Fig. 16-1 Pictorial of a CRT

The *horizontal deflection control circuits* provide the same functions as the vertical circuits, but in the horizontal direction.

The *time base generator supplies a voltage* which varies linearly with time, thus allowing us to view the vertical deflection as a function of time. This function is sometimes called the "sweep circuit" because it provides a means of sweeping the spot from side to side.

The *power supplies* provide voltages to operate the various circuits and also allow control of the spot focus and intensity.

The operational controls of most common types of oscilloscopes have the following functions:

1. The **Intensity** control regulates the brightness of the trace.
2. The **Vertical Position** control locates the starting point of the trace up and down on the face of the scope.
3. The **Vertical Gain Vernier** adjusts the size of the trace in the vertical direction.
4. The **Vertical Gain Selector** controls the amount of vertical displacement per input volt. This control must be calibrated before the oscilloscope is used to measure voltages. Calibration will be discussed later in this experiment.
5. The **Sync (Int-Ext)** switch determines whether the scope sweep is started by the input voltage (INT) position or by some external signal (EXT) position.
6. The **Sync Amplitude** control determines the value of voltage at which the trace starts.

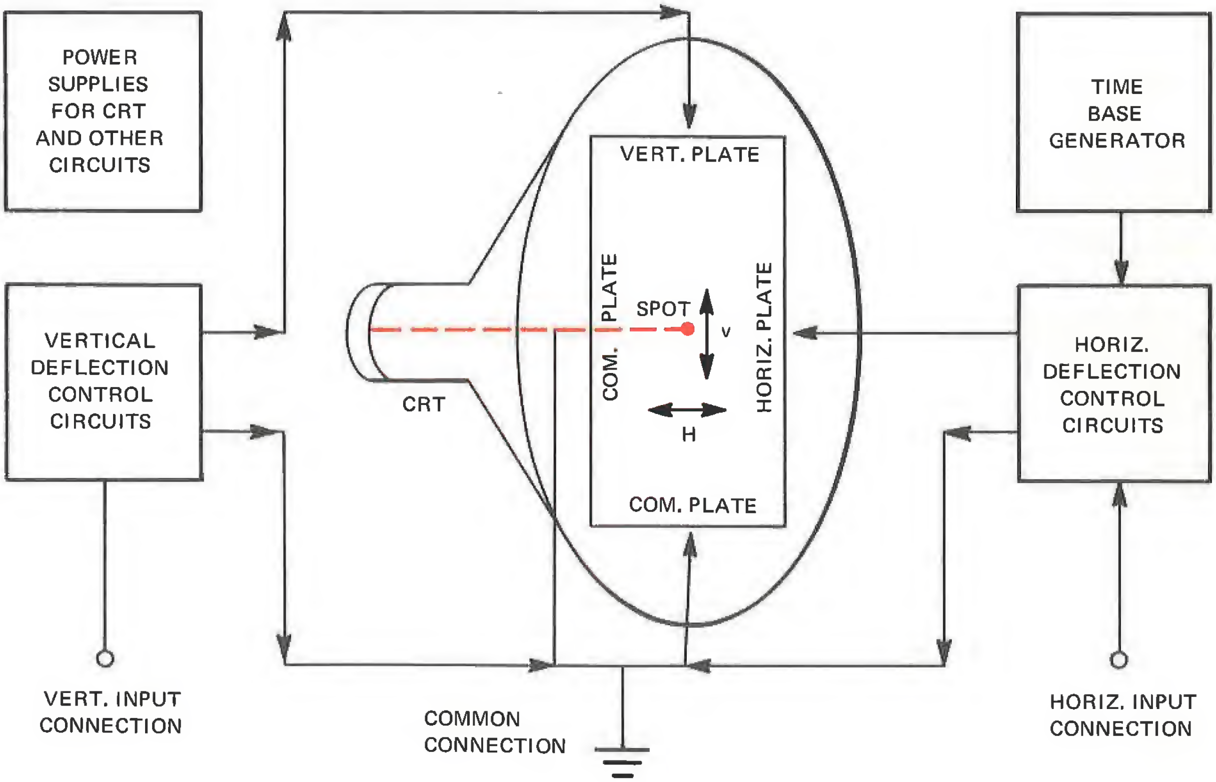


Fig. 16-2 Functional Diagram of a CRO

7. The **Horizontal Sweep Selector** is a coarse adjustment of the sweep rate.
8. The **Sweep Vernier Control** is a fine adjustment of the sweep rate.
9. The **Horizontal Gain** control adjusts the physical length of the trace.
10. The **Horizontal Position** control locates the position in the horizontal direction at which the sweep starts.
11. The **Focus** and **Astigmatism** control regulates the distinctness of the trace. It should be adjusted for the sharpest possible line on the scope face.

In general, the oscilloscope has three main functions:

1. The measurement of AC voltages
2. The measurement of time periods
3. The viewing of AC wave shapes

There are a number of more specialized uses of the oscilloscope. These, however, are beyond the scope of this experiment.

* Before AC voltages can be accurately measured with an oscilloscope, the vertical deflection must be calibrated. Basically, scope calibration is achieved by applying a known value of AC voltage to the vertical input and adjusting the **Vertical Gain Selector** and **Vernier** for a convenient amount of deflection. For example, suppose the scope face is marked off into squares which are 0.1 in. high and we apply an AC voltage to the vertical input; which we know as 10 volts, peak-to-peak. Now, if the **Vertical Gain Selector** and **Vernier** are adjusted until the trace on the face of the scope is exactly 2.0 inches high, the deflection sensitivity will be

$$\text{Deflection sensitivity} = \frac{\text{voltage}}{\text{deflection}} = \frac{10\ \text{Volts}}{2\ \text{in.}} = 5\ \text{volts/in.}$$

The scope in the example is now said to be calibrated. If we now apply an unknown value of voltage to the vertical input and observe a deflection, of say, 3.5 in., we can conclude that the unknown voltage has a peak-to-peak value of

$$E_{p\text{-}p} = (\text{deflection sensitivity})\ (\text{deflection}) = (5\ \text{volts/in.})\ (3.5\ \text{in.}) = 17.5\ \text{volts}$$

And we can say we have measured the unknown voltage.

Most oscilloscope manufacturers provide a voltage to be used in calibration via a front panel terminal. In some cases the calibration voltage is a square voltage wave form of an accurately known amplitude. In other cases the calibration voltage may be a 6.3 volt-rms (approximately 17.8 volts peak-to-peak) sine wave. In a few cases a calibrate (CAL) position is present on the **Vertical Gain Selector** switch. Such an arrangement allows the calibration voltage to be applied to the input by placing the switch in this position.

Once the scope is calibrated, the **Vertical Gain Vernier** setting **must not be altered** as this would change the calibration. The **Vertical Gain Selector** switch may be changed from position to position provided the deflection-sensitivity value is appropriately corrected for each new position.

It should also be noted that some oscilloscopes use a 10-to-1 probe. That is, the probe divides the input voltage by a factor of 10. If the scope is calibrated without the probe attached, subsequent voltage readings with the 10-to-1 probe must be corrected by multiplying them by 10.

Alternating current can be measured

*This material appears in a McGraw-Hill publication, EXPERIMENTS IN ELECTRICITY – ALTERNATING CURRENT, and is reproduced with permission of the publisher.

with an oscilloscope by first passing the current through a resistor of known value and then measuring the voltage drop across it. The current is then given by

$$I = \frac{E}{R}$$

In using the oscilloscope for time measurements we must calibrate the horizontal deflection as a function of time. To do this we switch **Horizontal Sweep Selector** to one of the internal sweep positions. A signal of known time duration is applied to the vertical input, the **Horizontal Sweep Selector** and **Horizontal Gain** controls are adjusted for a convenient time-to-displacement ratio.

For example, if a 60 Hz signal is used then the time of one complete alteration is

$$t = \frac{1}{60} \text{ sec} = 16.6 \text{ ms.}$$

If the sweep controls are set so that one cycle of the input voltage occupies 1.66 in. of horizontal distance, the horizontal deflection factor is

$$\text{Horizontal Deflection Factor} = \frac{\text{Time of one cycle}}{\text{Deflection of one cycle}} = \frac{16.6 \text{ ms}}{1.66 \text{ in.}} = 10 \text{ ms/in.}$$

If a signal of unknown duration is now observed to produce a complete cycle in 0.83 in., then the period of the signal is

$$t = (\text{Horiz. defl. factor})(\text{Horiz. sweep per cycle}) = (10 \text{ ms/in.})(0.83 \text{ in.}) = 8.3 \text{ ms}$$

And its frequency is

$$f = \frac{1}{t} = \frac{1}{8.3 \times 10^{-3}} = 120 \text{ Hz}$$

In addition to the two measurements discussed above, the oscilloscope is very valuable for general wave shape observation to detect irregularities in form. Calibration is not always required for this type of viewing.

MATERIALS

1 Oscilloscope
1 Multimeter (with P-P voltage ranges)
1 Audio signal generator

PROCEDURE

1. Turn on the scope and set the controls as indicated below:
   a. Intensity - full clockwise
   b. Vertical position - midrange
   c. Horizontal position - midrange
   d. Sync switch - Int
   e. Horizontal sweep selection - any sweep position

   A horizontal line should now be present on the scope face. If it is not, ask the instructor to check the scope.

2. Make the following adjustments:
   a. Set the vertical gain selector to a midrange position.
   b. Adjust the intensity and focus for a sharp horizontal line.
   c. Set the horizontal gain for a line length of about two-thirds of the width of the scope face.

d. Adjust the vertical and horizontal position controls until the trace is centered on the scope face.

3. Calibrate the vertical deflection of the oscilloscope. This step is very important. If you have difficulty, ask the instructor for assistance.

4. Prepare the scope for 60 Hz viewing as follows:

   a. Switch the **Horizontal Sweep Selector** to the position which includes 60 Hz.

   b. Readjust the **Horizontal Gain** and **Horizontal Sweep Vernier** such that one complete cycle of the calibration signal is spread across just 1.66 in. of the CRT face. It may be necessary to readjust the **Sync Amplitude** for a stable presentation.

5. Connect the circuit shown in figure 16-3.

6. Adjust the VOM for reading AC peak-to-peak (P-P) voltage.

7. Set the audio generator frequency to 60 Hz and the output level to about 20 percent of its full value.

8. Using the oscilloscope, measure and record the peak-to-peak voltage.

9. Measure the period as accurately as possible and record it as T in the data table.

10. From the period measurement, compute and record the frequency of the generator output.

11. Record the peak-to-peak generator voltage as indicated by the VOM.

12. Compute the percent difference between the two values of peak-to-peak voltage.

13. Compute the percent difference between the generator frequency setting and the value determined in step 10.

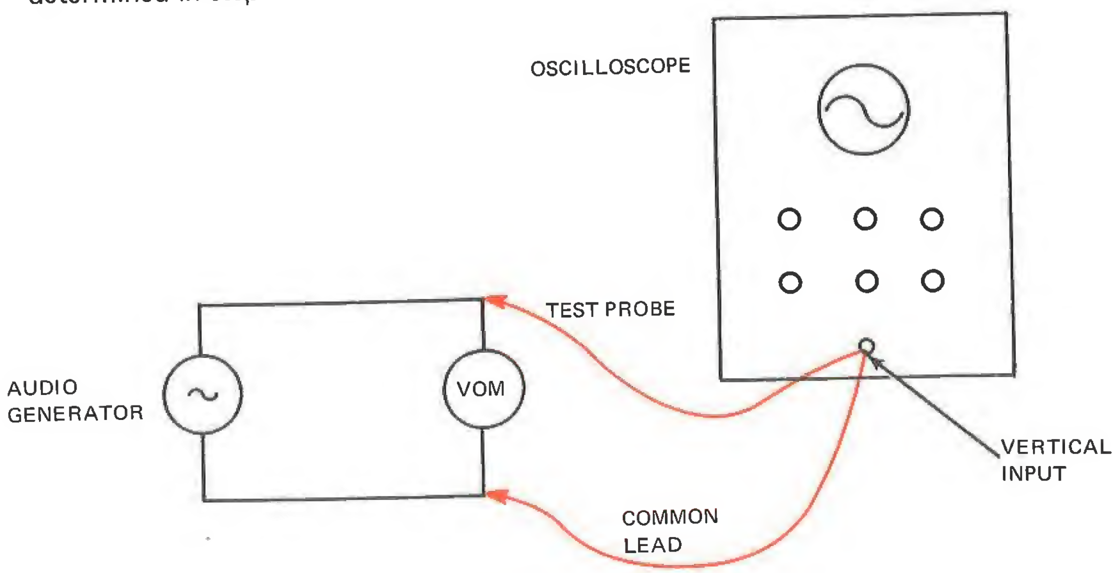


Fig. 16-3 The Experimental Circuit

14. Change the generator settings to 100 Hz and 30 percent output and repeat steps 8 through 13.

15. In similar manner, take data for generator settings of
    a. 120 Hz and 40% output.
    b. 150 Hz and 50% output.
    c. 200 Hz and 60% output.
    d. 250 Hz and 70% output.
    e. 400 Hz and 80% output.
    f. 600 Hz and 100% output.

Gen Freq.	Gen Output Level	$E_{p\text{-}p}$ (Scope)	T (Scope)	f (Scope)	$E_{p\text{-}p}$ (VTVM)	% Diff $E_{p\text{-}p}$	% Diff (f)
60 Hz	20%						
100 Hz	30%						
120 Hz	40%						
150 Hz	50%						
200 Hz	60%						
250 Hz	70%						
400 Hz	80%						
600 Hz	100%						

Fig. 16-4 The Data Table

ANALYSIS GUIDE. The primary objective of this experiment is to become familiar with the use of the oscilloscope to measure voltage and time. In analyzing the data, particular attention should be given to the extent to which this objective was satisfied.

PROBLEMS

1. A certain oscilloscope is calibrated to read 30 volts per cm with 10:1 probe. The horizontal deflection is calibrated to read 5 ms per cm. If the 10:1 probe is replaced with a 1:1 probe, what would be the amplitude and period of the voltage shown in figure 16-5?

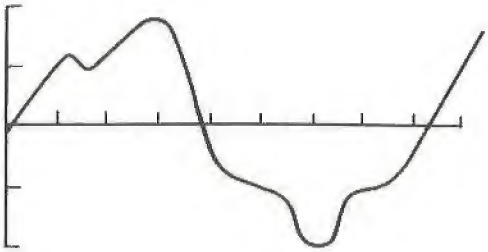


Fig. 16-5 Waveform for Problem 1

experiment 17 ALTERNATING CURRENT

INTRODUCTION. While unidirectional currents are very useful in operating electrical and electronic devices, they do present a number of distribution difficulties. As a result, virtually all electrical distribution is accomplished with alternating currents. In this experiment we shall examine some of the basic principles of *alternating current* circuit theory.

DISCUSSION. In the case of unidirectional currents, the flow is always in the same direction. This condition is caused by the fact that the polarity of a given DC source is always the same. An *alternating voltage* on the other hand goes through a repetitive cycle of polarity changes.

The most frequently encountered alternating voltage is in the form of a *sinusoid.* The variation of such a voltage with time is illustrated in figure 17-1. Such a voltage may be described by the equation

$$e = E_m \sin \omega t \qquad (17.1)$$

In equation 17.1, e is the value of the voltage at any time, t. E_m is the peak or maximum value. It should be noted that the *peak-to-peak* value is twice the value of E_m:

$$E_{p\text{-}p} = 2E_m \qquad (17.2)$$

ω is the *angular velocity* of the sinusoidal wave in *radians* per second. The value of ω is given by

$$\omega = 2\pi f \qquad (17.3)$$

where f is the *frequency* of the alternations in Hertz (cycles per second).

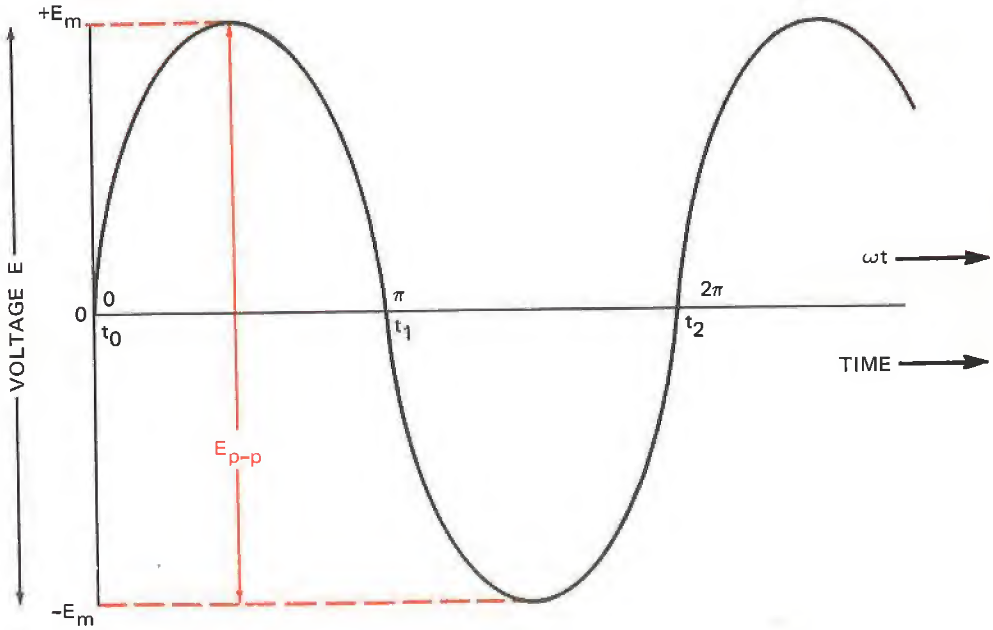


Fig. 17-1 A Sinusoidal Voltage

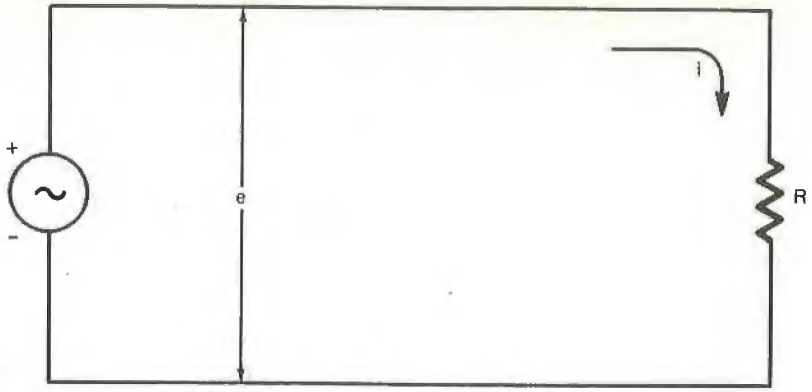


Fig. 17-2 A.C. Current in a Resistive Load

Occasionally we wish to deal with the length of the voltage wave directly in time units (seconds). In such a case, we refer to the time of one complete cycle as the *Period* of the wave. This length of time can be seen to be $T = t_2 - t_o$ in figure 17-1.

The time of one period is related to the frequency of alteration by

$$T = \frac{1}{f} \qquad \textbf{(17.4)}$$

If we connect an AC source to a resistive load, as indicated in figure 17-2, the instantaneous current that flows will be determined by Ohm's Law,

$$I = \frac{E}{R} \qquad \textbf{(17.5)}$$

If we substitute the value for e given in equation 17.1 into this relationship, the result is

$$I = \frac{E}{R} = \frac{E_m \sin \omega t}{R} = \frac{E_m}{R} \sin \omega t$$

which is, of course, a sinusoidal current having a peak value of E_m/R. That is,

$$I_m = \frac{E_m}{R} \qquad \textbf{(17.7)}$$

The equation for the current may, therefore, be written as

$$I = I_m \sin \omega t \qquad \textbf{(17.6a)}$$

The *instantaneous power* delivered to the load resistor can be found by the usual means:

$$p = EI = I^2R = \frac{E^2}{R} \qquad \textbf{(17.8)}$$

Substituting equation 17.1 and/or 17.6a into this power relationship gives us

$$p = \frac{E_m^2}{R} \sin^2 \omega t$$

From trigonometry we know that the sine squared of an angle is equal to one half of one minus the cosine of twice the angle. Therefore,

$$\sin^2 \omega t = \frac{1}{2} (1-\cos 2 \omega t)$$

and

$$p = \frac{E_m^2}{2R} (1-\cos 2 \omega t) = \frac{E_m^2}{2R} - \frac{E_m^2}{2R} \cos 2\omega t$$

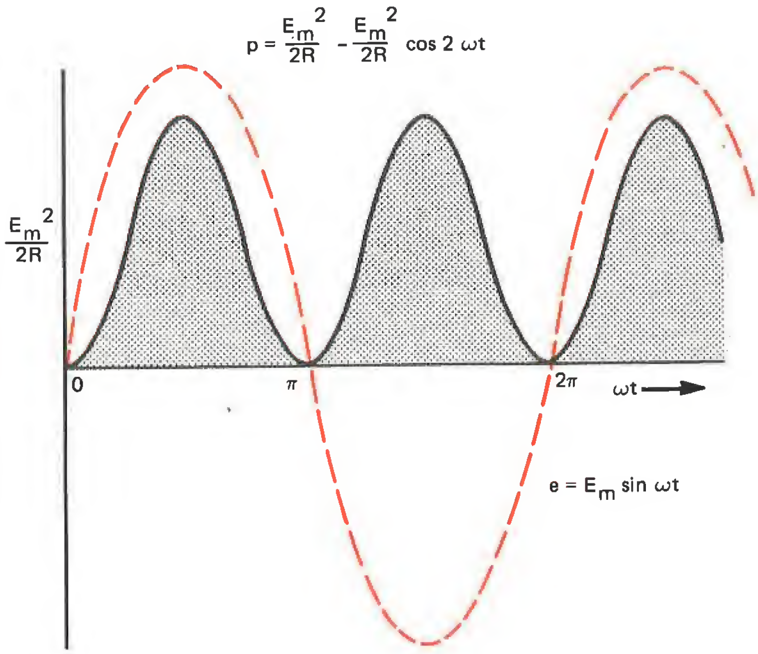


Fig. 17-3 Instantaneous Power

However, since it is the average power that generates the heat dissipated by the load resistor, we are more interested in average power than in instantaneous power. If we plot the instantaneous power versus time, the result will be as shown in figure 17-3. In taking the average of the power curve (shaded in fig. 17-3), we see that the average of the cosine term is zero if taken from $\omega t = 2\pi$. The average power is, therefore,

$$P = \frac{E_m^2}{2R} \qquad \textbf{(17.9)}$$

If we observe that the *maximum instantaneous* power is

$$P_m = E_m I_m = I_m^2 R = \frac{E_m^2}{R}$$

then the average power is related to this maximum instantaneous power by

$$\boxed{P = \frac{1}{2} P_m} \qquad \textbf{(17.10)}$$

For the sake of consistency, we would like the average power in an AC circuit to be related to the voltage and current in the same way as it is in a DC circuit. We therefore equate AC and DC power:

$$P_{dc} = P_{ac}$$

And if we substitute equation 17.10 for the AC power, we have

$$P_{dc} = \frac{1}{2} P_m$$

Now if we substitute I^2R and E^2/R for P_{dc} while substituting I_m^2 R and E_m^2/R for P_m, the results are

$$I^2R = \frac{1}{2} I_m^2 R$$

and

$$\frac{E^2}{R} = \frac{1}{2} \frac{E_m^2}{R}$$

Solving for I and E respectively gives

$$I = \frac{I_m}{\sqrt{2}} = 0.707\ I_m$$

and

$$E = \frac{E_m}{\sqrt{2}} = 0.707\ E_m \qquad (17.11)$$

In applying these quantities to AC circuits, we call them the *effective* or RMS current and voltage.

Most AC meters are calibrated to read these effective or RMS values. It is important to understand that these values are the ones which produce the same average power (heating effect) as an equal size DC voltage and current.

The average AC power dissipated by a resistor therefore becomes

$$P = I^2_{eff}R = \frac{E^2_{eff}}{R} = (E_{eff})\,(I_{eff}) \qquad (17.12)$$

where

$$I_{eff} = 0.707\ I_m \text{ and } E_{eff} = 0.707\ E_m$$

MATERIALS

1 Oscilloscope

1 Multimeter (VOM)

1 AC ammeter (0–0.5a)

1 Wattmeter (0–20 w)

1 Variable transformer

1 75-ohm resistor (20 watts)

PROCEDURE

1. Connect the circuit shown in figure 17-4.
2. Adjust the variable transformer for an *effective* voltage reading of 36 volts on the VOM.
3. Measure and record the peak-to-peak voltage with the oscilloscope.

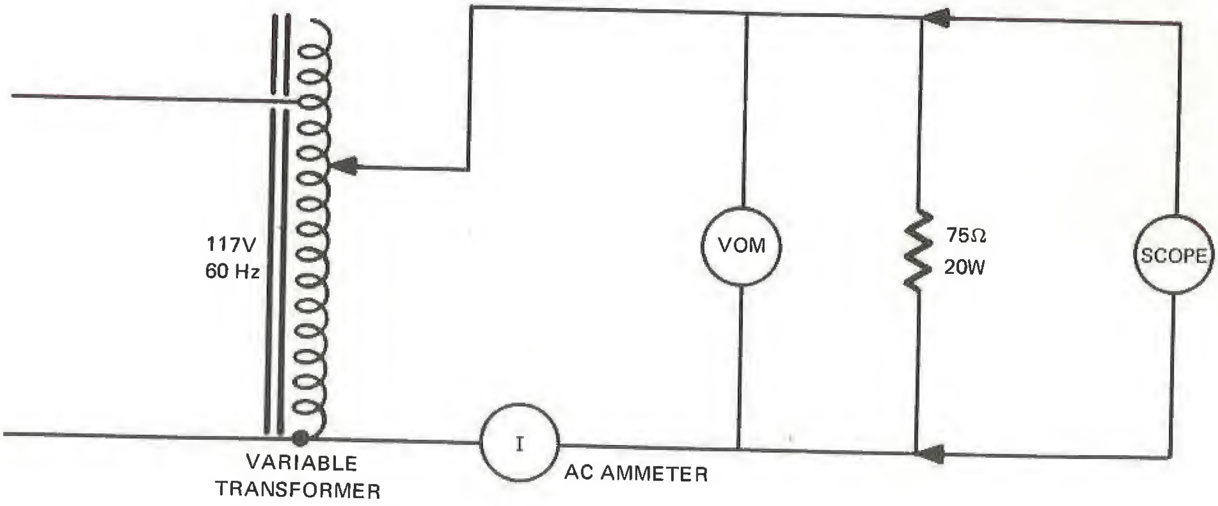


Fig. 17-4 The Experimental Circuit

E (VTVM)	R	E_{p-p} (scope)	E_m (scope)	E_m (VTVM)	% Diff E_m	I (meas)	I (comp)
36V							
32V							
28V							
24V							
20 V							

E (VTVM)	% Diff I	P (meas)	P (comp)	% Diff P	P_m (wattmeter)	P_m (comp)	% Diff P_m
36V							
32V							
28V							
24V							
20V							

Fig. 17-5 Data Table

4. Compute the value of E_m from the value of E_{p-p} as measured in step 3.
5. Compute E_m from the VOM value of effective voltage.
6. Compute the percent difference between the two values of E_m.
7. Record the effective circuit current as indicated by the ammeter.
8. Using the oscilloscope measurement and the resistance value, compute the effective current.
9. Compute the percent difference between two values of effective circuit currents.
10. Compute the average load power using the measured effective voltage and current.
11. Compute the maximum power using the oscilloscope measurement and the resistance value.
12. Replace the ammeter with the current coil of the wattmeter.
13. Replace the VOM with the voltage coil of the wattmeter.
14. Record the wattmeter reading (average power).

15. Compute the peak power using the wattmeter reading.
16. Compute the percent difference between two values of average power.
17. Compute the percent difference between the two values of peak power.
18. Repeat steps 2 through 17 for effective voltages of 32, 28, 24, and 20 volts.

ANALYSIS GUIDE. In the analysis of these data, you should consider primarily the extent to which the data agreed with the mathematical relationships between E_m, I_m, E_{eff}, P, and P_m.

PROBLEMS

1. A circuit like that shown in figure 17-4 has a resistance of 1kΩ and an input voltage of 168 sin 377t. What is the value of:
   a. The input frequency?
   b. The effective voltage?
   c. The instantaneous current?
   d. The effective current?
   e. The average power?
   f. The peak power?
2. If 50 sin 6280t volts is applied to a circuit in which 10^{-3} sin 6280t amps is flowing, what is the resistance of the circuit?

experiment 18 INDUCTIVE CIRCUITS

INTRODUCTION. *Inductors,* like capacitors and resistors, play a very important role in basic electricity. In this experiment we shall examine some of the basic characteristics of an inductor and the interrelationships between series and parallel inductances.

DISCUSSION. If we run an electric current through a coil of wire, a *magnetic field* will be established around the coil. If we now attempt to abruptly discontinue the current, the magnetic field will collapse, inducing a current into the coil in such a direction as to tend to prevent discontinuation of the current. This property of a coil of wire which tends to oppose any change in current flow is called *self inductance* of the coil. The extent of this self inductance in a given circuit is defined mathematically by

$$e = L\frac{di}{dt} \qquad (18.1)$$

where e is voltage across the coil in volts, L is the inductance in *henrys,* and di/dt is rate at which the current changes in amperes per second.

If two inductors are connected in series, as shown in figure 18-1, we may apply Kirchhoff's voltage law to the enclosed loop:

$$e_t = e_1 + e_2 \qquad (18.2)$$

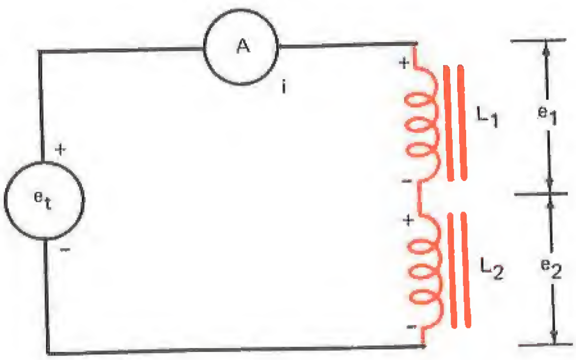


Fig. 18-1 Two Inductors in Series

From equation 18.1 we can observe that

$$e_1 = L_1\frac{di}{dt} \text{ and } e_2 = L_2\frac{di}{dt}$$

Substituting these expressions into equation 18.2 renders

$$e_t = L_1\frac{di}{dt} + L_2\frac{di}{dt}$$

or

$$e_t = \frac{di}{dt}(L_1 + L_2)$$

If we apply equation 18.1 to the whole circuit, we see that

$$e_t = L_t\frac{di}{dt}$$

Therefore,

$$L_t\frac{di}{dt} = \frac{di}{dt}(L_1 + L_2)$$

Canceling di/dt renders

$$L_t = L_1 + L_2 \qquad (18.3)$$

from which we conclude that the inductances in the series circuit may be added to determine the total circuit inductance.

Let us now turn our attention to a parallel inductive circuit, as shown in figure 18-2.

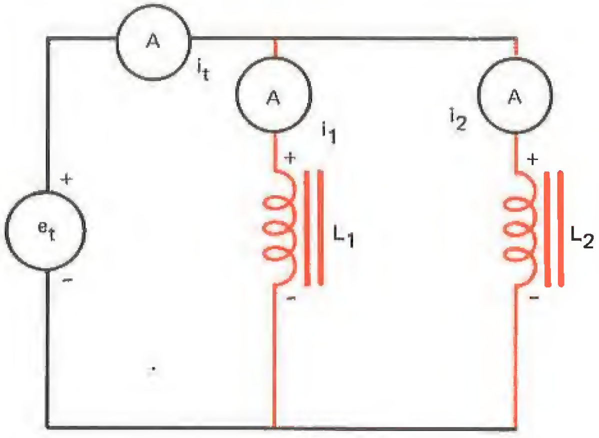


Fig. 18-2 Two Indicators in Parallel

In this case we observe that

$$i_t = i_1 + i_2$$

Moreover, if i_1 and/or i_2 changes in any way, that change must be reflected in i_t. Therefore, we may conclude that

$$\frac{di_t}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \quad \textbf{(18.4)}$$

Now if we solve equation 18.1 for di/dt, the result is

$$\frac{di}{dt} = \frac{e}{L}$$

Substituting this ratio into 18-4 for di_t/dt, di_1/dt and di_2/dt, respectively, renders

$$\frac{e_t}{L_t} = \frac{e_1}{L_1} + \frac{e_2}{L_2}$$

And since the inductors are simply in parallel, we may conclude that

$$e_t = e_1 = e_2$$

We may therefore substitute e_t into the equation above for e_1 and e_2:

$$\frac{e_t}{L_t} = \frac{e_t}{L_1} + \frac{e_t}{L_2}$$

Then canceling e_t in each term,

$$\boxed{\frac{1}{L_t} = \frac{1}{L_1} + \frac{1}{L_2}} \quad \textbf{(18.5)}$$

which reveals that inductances in parallel behave much like resistances in parallel.

In the special case of only two inductors in parallel, we may simplify (18.5) to

$$\boxed{L_t = \frac{L_1 L_2}{L_1 + L_2}} \quad \textbf{(18.6)}$$

which is the now familiar product over sum equation.

Up to this point we have considered only the case in which the magnetic field around one coil did not interfere in any way with the field around another inductor. Let us now turn to the condition in which the two magnetic fields do interact.

Consider the two coils shown in figure 18-3.

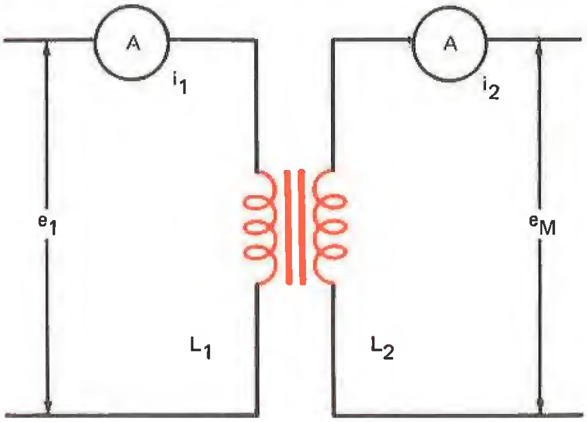


Fig. 18-3 Two Coupled Coils

We describe such a condition by saying that the magnetic fields of the coils are *coupled.*

If the current in one coil changes, it induces a voltage in the second coil. This effect is termed *mutual inductance* and is described mathematically by

$$e_M = M\frac{di}{dt} \qquad (18.7)$$

where M is the mutual inductance existing between the two coils. M is expressed in henrys as is self inductance.

This mutual inductance works both ways in the coupled coils: that is, if the current in the second coil changes, it induces a voltage into the first coil to the extent described by equation 18.7.

Let us now consider two coupled coils connected in series, as shown in figure 18-4.

By Kirchhoff's Law

$$e_t = e_1 + e_2$$

must apply. However, e_1 is now composed of two components, one based on equation 18.1 and the other based on equation 18.7.

That is,

$$e_1 = L_1\frac{di}{dt} \pm M\frac{di}{dt}$$

Similarly,

$$e_2 = L_2\frac{di}{dt} \pm M\frac{di}{dt}$$

The ± sign is determined by the direction in which the turns of wire on L_1 and L_2 are wound. If both are wound in the same direction, the signs are (+); but if they are wound in opposite directions, the signs are (–).

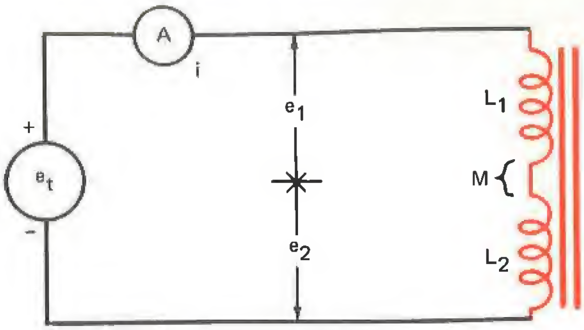


Fig. 18-4 Coupled Coils in Series

Substituting these values in the Kirchhoff's law equation and also substituting $e_t = L_t\frac{di}{dt}$ renders

$$L_t\frac{di}{dt} = L_1\frac{di}{dt} \pm M\frac{di}{dt} + L_2\frac{di}{dt} \pm M\frac{di}{dt}$$

which reduces algebraically to

$$L_t = L_1 + L_2 \pm 2M \qquad (18.8)$$

If the sign of 2M is (+), we say the circuit is connected in series aiding; if the sign of 2M is (–), we say the circuit is connected in series opposing.

Inductances are readily measured using equation (18.1)

$$e = L\frac{di}{dt}$$

If we apply a sinusoidal voltage and the magnetic field does not reach saturation, then di/dt becomes equal to

$$\frac{di}{dt} = \omega I_m$$

where ω is the angular velocity ($2\pi f$) and I_m is the peak current. Substituting this value into equation 18.1 gives us

$$L = \frac{E}{\omega I} \qquad (18.9)$$

E and I *can* be the effective values of voltage and current.

Measuring mutual inductance (M) is somewhat more involved. However, if we connect the coils in series aiding, as in figure 18-5, and apply equation 18.9, the result is

$$L_1 + L_2 + 2M = \frac{E_1}{\omega I_1}$$

If we now reverse one coil, the equation becomes

$$L_1 + L_2 - 2M = \frac{E_2}{\omega I_2}$$

Subtracting the two equations gives us

$$4M = \frac{E_1}{\omega I_1} - \frac{E_2}{\omega I_2}$$

or

$$M = \frac{1}{4\omega}\left(\frac{E_1}{I_1} - \frac{E_2}{I_2}\right) \qquad \textbf{(18.10)}$$

which allows us to determine M fairly easily.

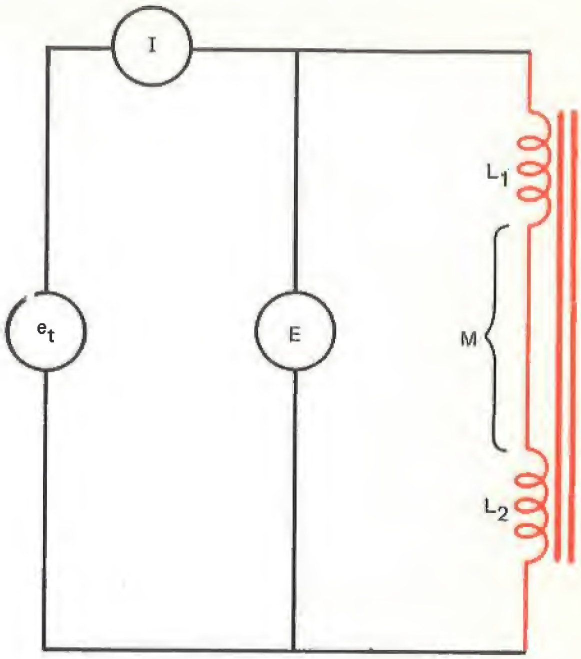


Fig. 18-5 First Step in Measuring M

MATERIALS

- 2 14-henry high Q inductors
- 1 Variable transformer
- 1 Multimeter
- 1 Transformer (1:1 turns ratio)
- 1 100-ohm resistor
- 1 Ammeter (AC) (0–50ma, 0–1a)

PROCEDURE

1. Connect the circuit shown in figure 18-6 using one of the 14-henry inductors.
2. Set the variable transformer output for an effective voltage (E_A) across the 100-ohm resistor of one volt. Compute and record the circuit I, in Part 1 of the data table.

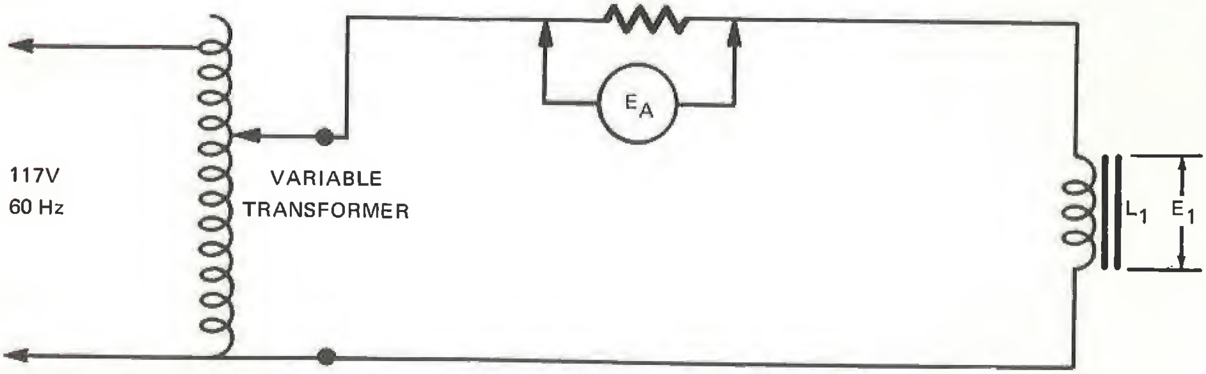


Fig. 18-6 The Experimental Circuit

3. Reconnect the voltmeter to measure the voltage across the inductor (E_1) and record this value.
4. Using equation 18.9, compute and record the value of L_1.
5. Replace the inductor with the other 14 H inductor (L_2) and repeat steps 1 through 4. Record the values as I_2, E_2, and L_2 in the data table.
6. Connect the two inductors in series and physically separate them by about 12 inches.
7. Repeat steps 1 through 4 using the two series inductors as L. Record the data as I_s, E_s, and L_s in the data table.
8. Reverse the connections of *one* coil and repeat steps 6 and 7 recording the data as I'_s, E'_s and L'_s.
9. Connect the two inductors in parallel and physically separate them by about 12 inches.
10. Repeat steps 1 through 4 using the parallel inductors as L. However, in step 2 use $E_A = 2$ volts instead of 1 volt. Record the data as I_p, E_p and L_p in the data table.
11. Identify one of the transformer windings and connect the circuit shown in figure 18-6. Replace the 100-ohm resistor with the AC ammeter and connect the VOM across the inductance.
12. Adjust the variable transformer for a *readable* current on the ammeter. As before, record the values of I_1, E_1 and L_1 in the data table (Part 2).
13. Identify the other transformer windings and repeat steps 11 and 12 recording the data as I_2, E_2 and L_2. (*Note:* E_2 *and* E_1 *should be nearly equal, to insure meaningful results.)*
14. Connect the two windings in series and repeat steps 11 and 12 recording the data as I_s, E_s, and L_s.
15. Reverse *one* of the transformer coils and repeat steps 11 and 12 recording the data as I'_s, E'_s, and L'_s.
16. With the values found in steps 4 and 5 and equation 18.3, compute L_s and record it in the data table (Part 3).
17. Compute the percent difference between this last value of L_s and that found in step 7.
18. Similarly, compute the percent difference between L_s (step 16) and L'_s (step 8).
19. With the values of L_1 and L_2 (steps 4 and 5), compute L_p using equation 18.5.
20. Compute the percent difference between L_p (step 19) and L_p (step 10).
21. With the values measured in steps 14 and 15, compute M using equation 18.10. Record this value in the data table, Part 4.
22. Using the value of M, L_1 and L_2 (steps 12 and 13) and equation 18.8, compute the values of L_s (comp) and L'_s (comp) and record them in the data table.
23. Compute the percent differences between the values found in step 22 and those found in steps 14 and 15.

I_1	E_1	L_1	I_2	E_2	L_2	I_s	E_s	L_s	I'_s	E'_s	L'_s	I_p	E_p	L_p

Part 1 (No Coupling)

I_1	E_1	L_1	I_2	E_2	L_2	I_s	E_s	L_s	I'_s	E'_s	L'_s

Part 2 (Coupled)

L_s (Comp)	% Diff L_s, L_s	% Diff L_s, L'_s	L_p (Comp)	% Diff L_p, L_p

Part 3 (No Coupling)

M	L_s (Comp)	% Diff L_s, L_s	L'_s (Comp)	% Diff L_s, L'_s

Part 4 (Coupled)

Fig. 18-7 The Data Table

ANALYSIS GUIDE. In considering the results of this experiment, you should give particular attention to the extent to which the equations given in the discussion predicted the measured results.

PROBLEMS

1. Three inductors of 10, 8, and 5 henrys are connected in series with no coupling between them. What is the total inductance?
2. What are the two possible values of total inductance in figure 18-8?
3. What would be the inductance values in Problem 2 if L_2 were in parallel with the series combination of L_1 and L_2?

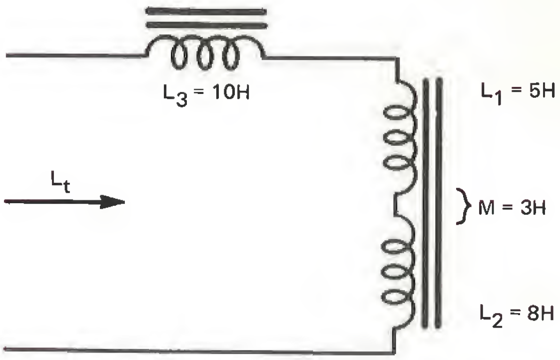


Fig. 18-8 Circuit for Problem 2

experiment 19 REACTIVE CIRCUITS

INTRODUCTION. As an alternating current is passed through a capacitor or inductor, a form of opposition is encountered which is independent of the resistance of the element. In this experiment we shall examine the characteristics of this form of opposition.

DISCUSSION. The opposition mentioned in the objective is called *reactance* (X). In the case of an inductor it is called *inductive reactance* (X_L), while for a capacitor it is called *capacitive reactance* (X_C).

The instantaneous voltage across an inductor is given by

$$e = L\frac{di}{dt}$$

where e is the voltage, L is the inductance, and di/dt is the rate at which the current changes.

The magnitude of di/dt for a sinusoidal current is

$$\frac{di}{dt} = 2\pi f I_m$$

We may, therefore, write the voltage equation as

$$E = 2\pi f L I$$

where E and I may be peak, peak-to-peak, average, or RMS values.

In order to be consistent with Ohm's Law, we define the inductive reactance as

$$X_L = \frac{E}{I} \text{ ohms} \qquad (19.1)$$

and we see that

$$X_L = 2\pi f L \text{ ohms} \qquad (19.2)$$

or, since $\omega = 2\pi f$,

$$X_L = \omega L \text{ ohms} \qquad (19.3)$$

Equation 19.2 indicates that the value of the inductive reactance is directly proportional to the inductance and to frequency of the current. Figure 19-1 shows a plot of the value of X_L versus frequency.

Since we have defined X_L in terms of the ratio of inductor voltage to inductor current, the units in which X_L is expressed are volts-per-amp or *ohms*.

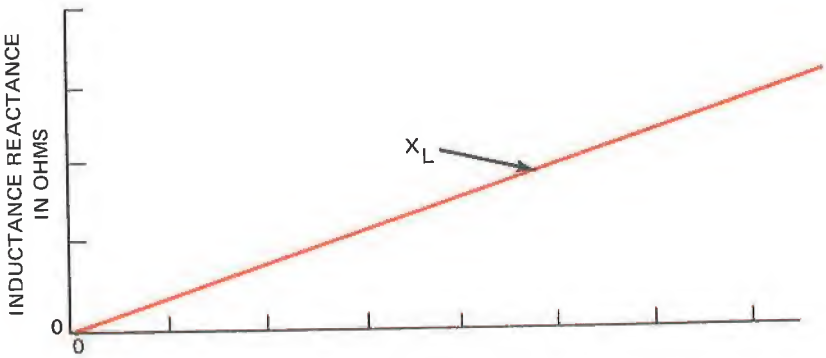


Fig. 19-1 Inductive Reactance versus Frequency

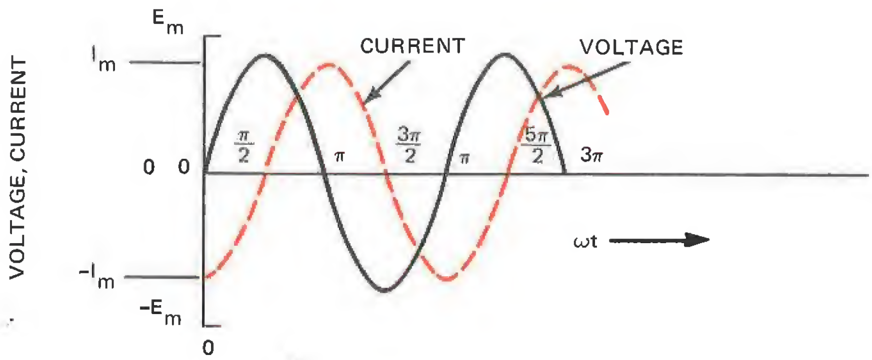


Fig. 19-2 Voltage and Current in an Inductor

Returning to the original equation

$$e = L\frac{di}{dt}$$

we observe that

$$\frac{di}{dt} = \frac{e}{L}$$

In other words, the rate of change of the current is directly proportional to the voltage. If the voltage is a sine wave, then the current will be undergoing the greatest rate of change when the voltage is at maximum. Figure 19-2 shows a plot of voltage and current which satisfies this condition.

From this plot we see that the voltage and current are $\pi/2$ radians (or 90°) apart, and that the current is lagging behind the voltage.

This *phase angle* is dependent upon the frequency and, for an RL circuit, it can be written

$$\theta_1 = \tan^{-1}\frac{X_L}{R} \qquad (19.4)$$

This phase shift between the voltage and current is characteristic of an inductive reactance and we usually write equations 19.1, 19.2, and 19.3 in the forms

$$X_L = +j\frac{E}{I} = +j\,2\pi fL = +j\omega L \qquad (19.5)$$

where the +j indicates the presence of the 90° phase shift between the voltage and current.

In the case of an AC current in a capacitor, we may write

$$i = C\frac{de}{dt}$$

where i is the current, C is the capacitance and de/dt is the rate of change of the voltage with time.

Following the same argument as used with the inductor, we see that the magnitude of de/dt for a sinusoidal current will be

$$\frac{de}{dt} = 2\pi fE_m$$

and

$$I = 2\pi fCE$$

from which we have

$$\frac{E}{I} = \frac{1}{2\pi fC}$$

and, as before, we define the capacitive reactance to be

$$X_C = \frac{E}{I} = \frac{1}{2\pi fC} = \frac{1}{\omega C}\text{ ohms} \qquad (19.6)$$

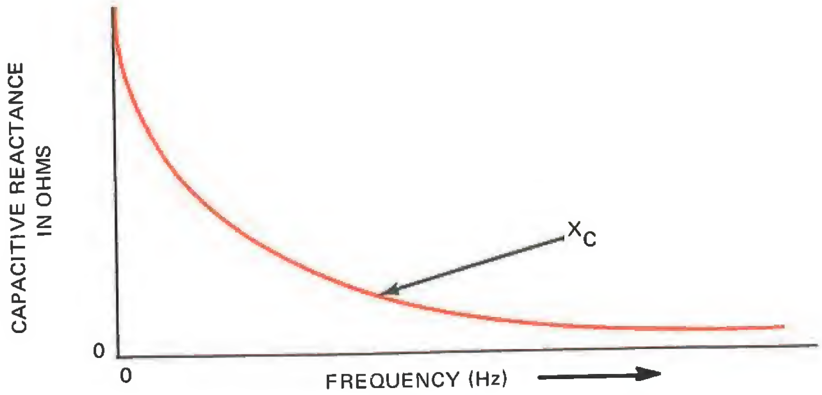


Fig. 19-3 Capacitive Reactance versus Frequency

However, in this case we see that the value of X_C varies inversely with C and f. A plot of the X_C versus frequency would appear as shown in figure 19-3.

Referring again to the equation

$$i = C\frac{de}{dt}$$

we observe that

$$\frac{de}{dt} = \frac{i}{C}$$

The rate of change of the voltage is therefore directly proportional to the value of the current. Figure 19-4 shows plots of voltage and current which satisfy this condition.

As in the case of inductive reactance, there is a $\pi/2$ radian or 90° phase shift. However, in this case the current *leads* the voltage by 90°. To account for the phase shift, we usually write equation 19.5 as

$$X_C = -j\frac{E}{I} = -j\frac{1}{2\pi fC} = -j\frac{1}{\omega C}$$

The phase angle is again dependent upon the frequency and, for a series RC circuit, it is written

$$\theta_C = \tan^{-1}\frac{X_C}{R} \qquad \textbf{(19.7)}$$

Measurements of reactance magnitudes may be readily made by measuring voltage and current and applying equation 19.1 or 19.5.

The phase shift associated with the reactances calls for a somewhat more involved measurement technique. Consider the oscilloscope pictorial shown in figure 19-5.

Let us suppose that we apply two voltages, E_v and E_h, to the vertical and horizontal inputs of the oscilloscope:

$$E_v = E_1 \sin \omega t$$

and

$$E_h = E_2 \sin (\omega t - \theta)$$

These two voltages have different magnitudes and are displaced from each other by the phase angle θ.

The vertical and horizontal deflection of the spot on the CRT screen will be

$$y = E_v \sin \omega t$$

and

$$x = E_h \sin (\omega t - \theta)$$

respectively. E_v and E_h are the control constants introduced by the vertical and horizon-

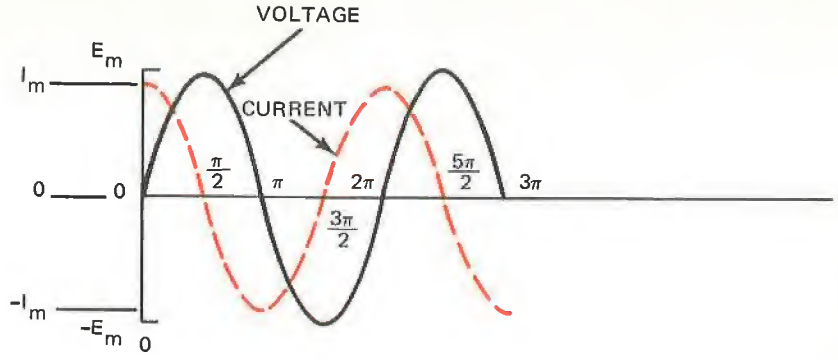


Fig. 19-4 Voltage and Current in a Capacitor

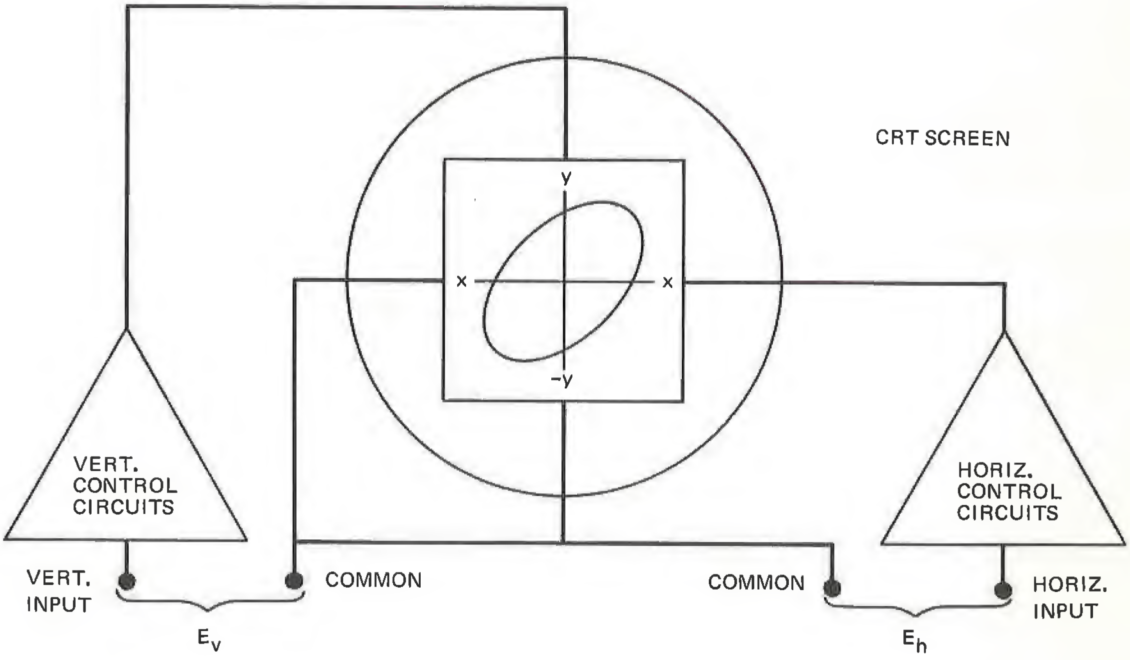


Fig. 19-5 Phase Measurement with an Oscilloscope

tal control circuits of the oscilloscope times the peak input voltages, E_1 and E_2. If we solve these two equations simultaneously and eliminate the factor ωt, the result is

$$\frac{x^2}{E_h^2} - \frac{2xy \cos \theta}{E_h E_v} + \frac{y^2}{E_v^2} = \sin^2 \theta$$

This is the equation of an ellipse whose tilt is a function of the phase angle θ. Figure 19-6 shows a sketch of one such ellipse.

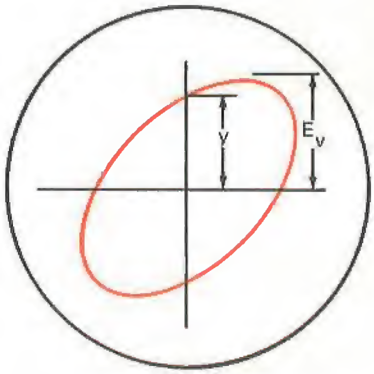


Fig. 19-6 Ellipse on CRT Screen

Referring to the equation for the ellipse, we see that when x = 0 it reduces to

$$\frac{y^2}{E_V^2} = \sin^2\theta \qquad \text{or} \qquad \boxed{\sin\theta = \frac{y}{E_V}} \qquad (19.8)$$

This equation allows us to determine the angle by measuring the distances y and E_V on the oscilloscope. These values are then substituted into equation 19.6 and the value of θ computed.

In a practical case, we are most often interested in the phase angle between the circuit voltage and current. We must, therefore, convert the circuit current to a voltage for oscilloscope viewing. This can be readily accomplished by placing a resistor in series with the circuit under investigation. The size of this series resistor must be small enough, compared to the circuit values, that it does not appreciably alter the circuit current.

MATERIALS

- 1 Audio generator
- 1 Multimeter
- 1 Oscilloscope (with horizontal input)
- 1 1-μF capacitor
- 1 1-H inductor
- 1 75-ohm resistor
- 1 Sheet linear graph paper

PROCEDURE

1. Disconnect the ground link on the audio generator and assemble the circuit shown in figure 19-7 using the balanced generator output terminals.

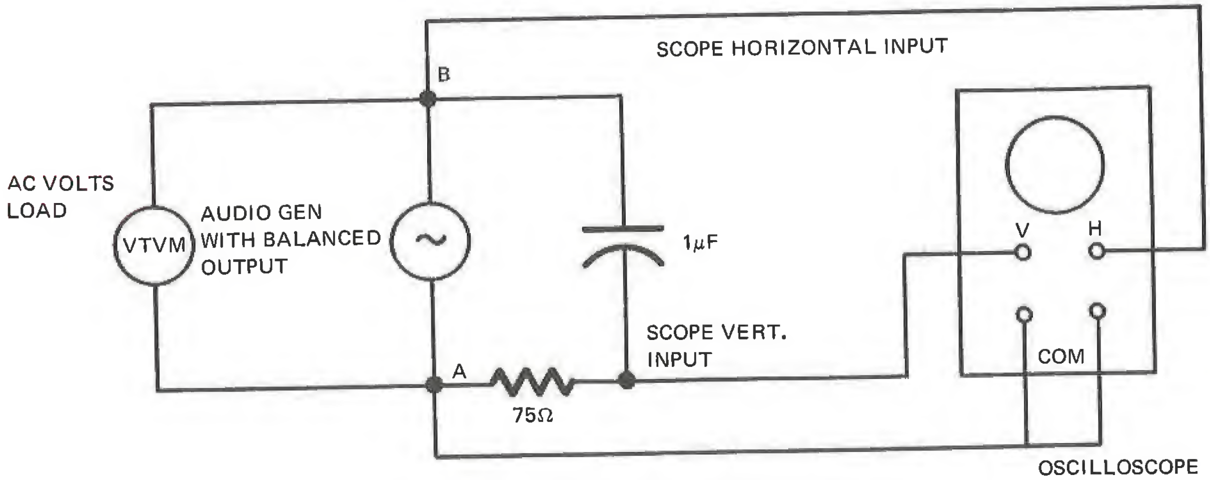


Fig. 19-7 The Experimental Circuit

Use particular care to connect the instrument common leads as shown in the figure.

2. Set the audio generator for an output frequency of 25 Hz.
3. Adjust the audio generator output level for a VOM reading of 6 volts (effective). This value is E_I. Keep this value constant throughout the collecting of data.

4. Move the AC volts lead of the VOM from point B to point D and record the voltage (E_R) across the 75-ohm resistor. Return the lead to point B when this measurement is completed.

5. Compute and record the circuit current and the capacitor voltage drop, E_C.

6. Using the voltage and current, compute and record the value of the reactance as a measured quantity.

7. Using equation 19.2 (or 19.6) and the generator frequency, compute and record the value of the reactance (computed).

8. Compute the percent difference between the two values of reactance.

9. Using the vertical and horizontal gain controls, adjust the oscilloscope for a viewable ellipse on the screen. Read and record the distances y and E_V from the ellipse.

10. Using equation 19.8, compute and record the value of sin θ.

11. Compute and record the value of the phase angle between the voltage and current (θ).

12. Change the audio generator frequency to 50 Hz and repeat steps 3 through 11.

13. In like manner, repeat steps 3 through 11 for frequencies of 100, 200, 300, 400, 500, 600, 700, 800, 900 and 1000 Hz.

14. Remove the capacitor from the experimental circuit and replace it with the inductor.

15. Repeat steps 3 through 13 using the inductor. Record the data in Part 2 of the data table (figure 19-8).

16. On a single sheet of linear graph paper, plot the values of the measured reactance versus frequency.

ANALYSIS GUIDE. In the analysis of these data, you should consider the extent of agreement between the equations given in the discussion and your experimental results.

In particular, consider the reasons for any variation in phase angle with frequency.

PROBLEMS

1. At what frequency is the reactance of a 0.5 μF capacitor equal to that of a 0.5 H inductor?

2. A certain circuit element is known to be either a pure capacitance or a pure inductance. If the reactance of the element is found to be 397.5KΩ and 318KΩ at 400 Hz and 500 Hz, respectively, which is it? What is its value?

E_I	H_z	E_c	E_R	I	X_C (meas)	X_C (comp)	% Diff X_C	y	E_v	Sin θ	θ
6V	25										
6V	50										
6V	100										
6V	200										
6V	300										
6V	400										
6V	500										
6V	600										
6V	700										
6V	800										
6V	900										
6V	1000										

E_I	H_z	E_L	E_R	I	X_L (meas)	X_L (comp)	% Diff X_L	y	E_v	Sin θ	θ
6V	25										
6V	50										
6V	100										
6V	200										
6V	300										
6V	400										
6V	500										
6V	600										
6V	700										
6V	800										
6V	900										
6V	1000										

Fig. 19-8 The Data Table

experiment 20 SERIES IMPEDANCES

INTRODUCTION. In the general case, the opposition offered to an AC current by a circuit is a complex quantity. In this experiment we shall examine the manner in which resistances and reactances are combined in a series circuit to provide this complex impedance.

DISCUSSION. In our number system there are both *real* and *imaginary numbers.* Moreover, these two types of numbers may be combined to form *complex numbers.* Figure 20-1 illustrates graphically the interrelation between real, imaginary, and complex numbers. From this diagram we should observe that the real numbers and the imaginary ones do not overlap. The complex numbers, however, include both and all possible combinations of both.

The opposition that a circuit offers to an AC current is exactly like the number system. It includes real opposition (resistance), imaginary opposition (reactance), and complex opposition (impedance). Graphically, after the pattern of figure 20-1, the interrelation of these various types of opposition can be represented as shown in figure 20-2. As in the case of the numbers, the resistances (real quantities) and reactances (imaginary quantities) are independent of each other and do not overlap. The complex impedance includes both the resistances and reactances as well as all possible combinations of resistance and reactance.

In this experiment we shall consider only series combinations of resistances and reactances. There are, of course, four conceivable combinations:

1. Resistance and capacitive reactance
2. Resistance and inductive reactance
3. Inductive reactance and capacitive reactance
4. Resistance, inductive reactance, and capacitive reactance

Because any practical inductor has some winding resistance, the third of these possibilities does not arise in practicality. Let us consider the remaining three possibilities in turn.

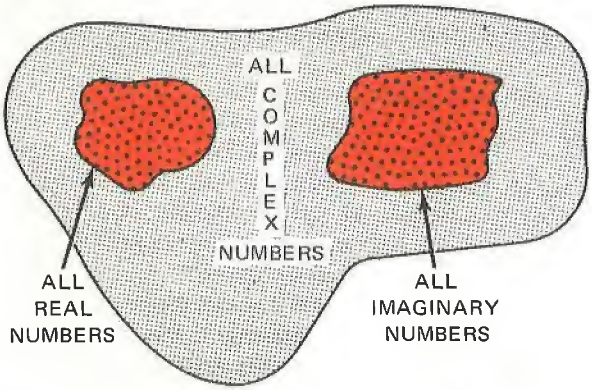


Fig. 20-1 The Imaginary Numbers

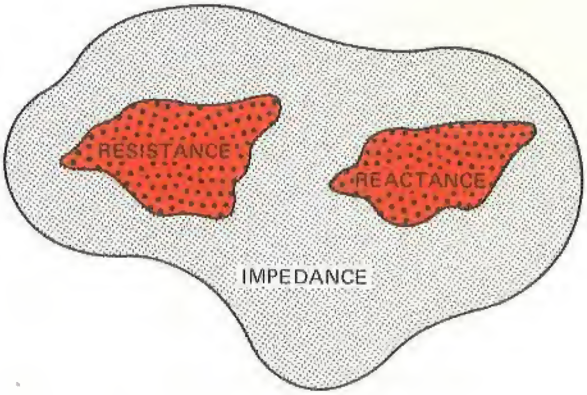


Fig. 20-2 Electrical Impedance

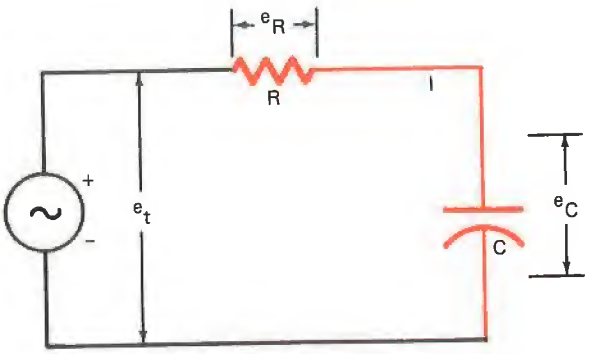


Fig. 20-3 A Series RC Circuit

Consider the series RC circuit shown in figure 20-3. By applying Kirchhoff's law, we see that

$$e_t = e_R + e_C \qquad (20.1)$$

Since the voltages across the resistor and capacitor are

$$e_R = iR \quad \text{and} \quad e_C = i(-jX_C)$$

the total voltage is therefore

$$e_t = iR + i(-jX_C)$$

And if we define the circuit impedance (Z) according to Ohm's Law, $Z = e_t/i$, we may write

$$iZ = iR + i(-jX_C)$$

But since the current is the same throughout a series circuit, we can cancel the current factor in each term. The impedance is, therefore,

$$\boxed{Z = R - jX_C} \qquad (20.2)$$

If we plot this equation on a set of real-imaginary coordinates, the result is as shown in figure 20-4.

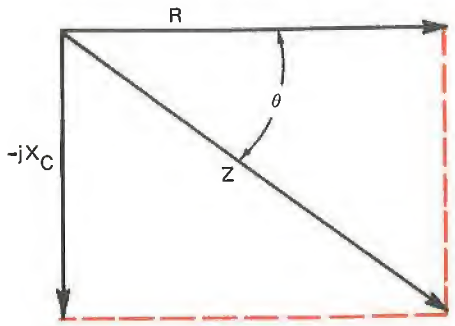


Fig. 20-4 Impedance of an RC Circuit

It is apparent from this diagram that we can write the impedance in polar form as

$$R - jX_C = Z\angle{-\theta} \qquad (20.3)$$

It should be evident from the above discussion that the impedance is a vector quantity and must be treated accordingly in any calculations.

Analysis of the RL circuit shown in figure 20-5 is very similar to that of the RC circuit just discussed.

The total voltage will be

$$e_t = e_R + e_L$$

Therefore

$$iZ = iR + i(+jX_L)$$

and

$$Z = R + jX_L \qquad (20.4)$$

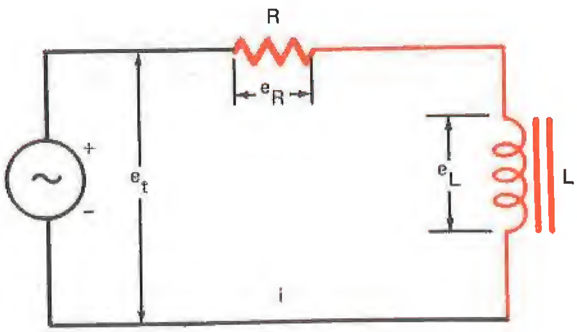


Fig. 20-5 A Series R_L Circuit

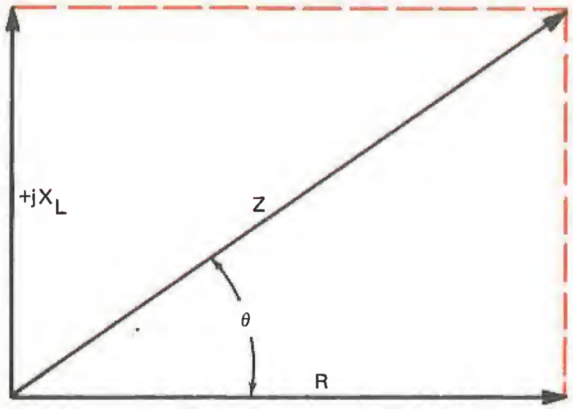


Fig. 20-6 Impedance of an R_L Circuit

A vector diagram of the two components and the impedance would be as shown in figure 20-6. From this diagram we see that we can write the impedance as

$$Z\angle\theta = R + jX_L \quad (20.5)$$

In a practical inductive circuit, at least some part of the circuit resistance would be the winding resistance of the coil.

Both of the cases discussed so far involve a single reactive component which is frequency dependent. If we plot the current opposition versus frequency for each, the result would be as shown in figure 20-7.

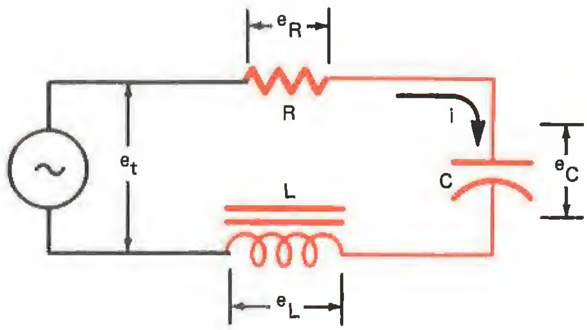


Fig. 20-8 An RLC Circuit

The third practical combination is shown in figure 20-8. The voltage loop equation for this circuit is

$$e_T = e_R + e_C + e_L$$

which can be rewritten as

$$iZ = iR + i(-jX_C) + i(+jX_L)$$

From which we see that the impedance is

$$Z = R + jX_L - jX_C \quad (20.6a)$$

or

$$Z = R + j(X_L - X_C) \quad (20.6b)$$

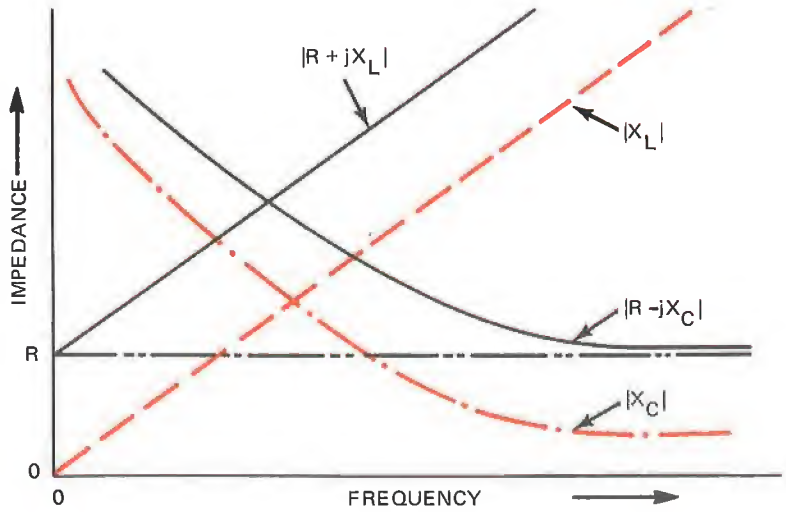


Fig. 20-7 Impedance Versus Frequency, RL and RC Circuit

The vector diagram of the impedance in this case is as shown in figure 20-9.

On the basis of this pictorial we observe that the impedance may be written as

$$Z\angle\pm\theta = R + j(X_L - X_C) \qquad (20.7)$$

We should observe that the reactive component of this impedance is equal to the difference between X_L and X_C and may, therefore, be either inductive, capacitive, or zero depending on the frequency. A plot of impedance versus frequency (figure 20-10) shows the various possible conditions.

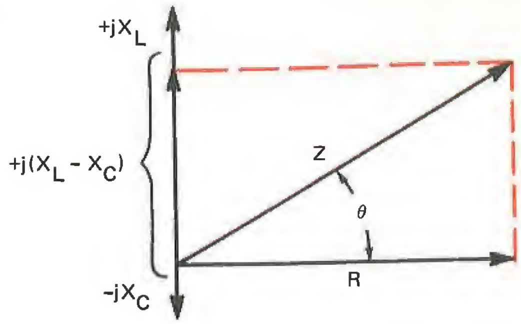


Fig. 20-9 Impedance of an RL Circuit

Notice that at the point f_o the reactive component is zero and $Z = R$. Above f_o the circuit is inductive and below f_o it is capacitive.

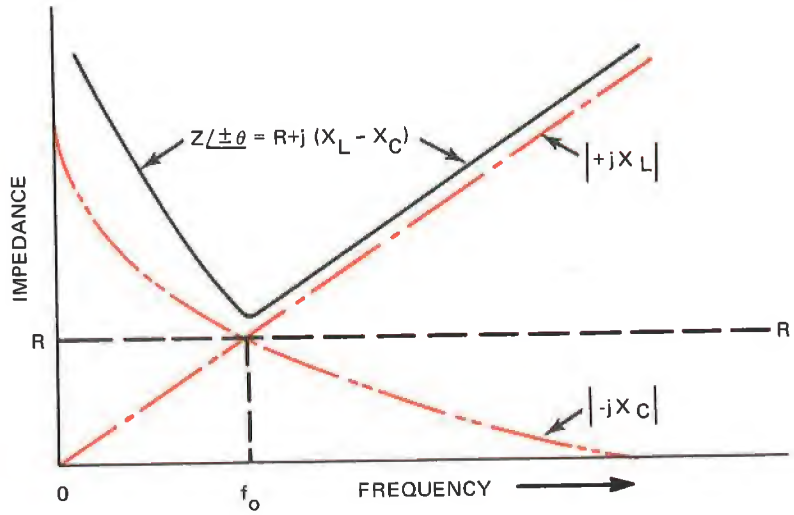


Fig. 20-10 Impedance vs. Frequency, RLC Circuit

MATERIALS

- 1 1-μF capacitor
- 1 1-H inductor
- 1 250-ohm resistor
- 1 Audio generator
- 1 Multimeter
- 1 Oscilloscope
- 3 Sheets of linear graph paper

PROCEDURE

1. Measure and record the DC resistance (R_c) of the inductor.
2. Assemble the circuit shown in figure 20-11.

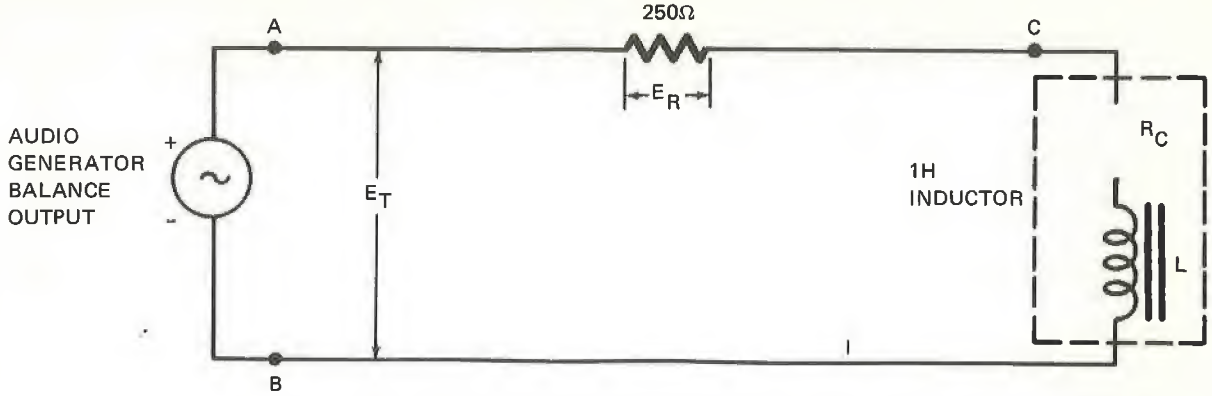


Fig. 20-11 The Experimental Circuit

3. Disconnect the ground strap so that the generator is operating with a balanced output.
4. Connect the VOM to measure the effective voltage across the 250-ohm resistor.
5. Set the generator controls for an output frequency of 50 Hz and a voltage of about 0.5 volts across the 250-ohm resistor. Record this value of voltage (E_R) and maintain it throughout the following steps.
6. Compute and record the circuit current (I).
7. Measure and record the generator output voltage (E_T).
8. Using the circuit current and voltage, determine the circuit impedance magnitude (Z_{meas}).

RL Circuit Data				E_R =			R_C =		
f Hz	E_T	I	Z Meas	θ Meas	X_L Comp	Z Comp	θ Comp	% Diff Z	% Diff θ
50									
100									
150									
200									
250									
300									
400									
500									
600									

Fig. 20-12 The Data Tables

RC Circuit Data				E_R =					
f Hz	E_T	I	Z Meas	θ Meas	X_C Comp	Z Comp	θ Comp	% Diff Z	% Diff θ
50									
100									
150									
200									
250									
300									
400									
500									
600									

RLC Circuit Data				E_R =				R_C =		
f Hz	E_T	I	Z Meas	θ Meas	X_L Comp	X_C Comp	Z Comp	θ Comp	% Diff Z	% Diff θ
50										
100										
150										
200										
300										
400										
500										
600										

Fig. 20-12 Data Tables (continued)

9. Connect the oscilloscope common lead to point A, the vertical input to point B, and the horizontal input to point C. Adjust the scope for an ellipse on the screen.

10. Using the oscilloscope pattern, determine and record the impedance angle (θ). This is the phase angle between the circuit current and voltage.

11. Using the generator frequency and the component values, compute and record the value of the circuit reactance(s) (X_L and/or X_C).

12. Using the total circuit resistance and reactance in the appropriate equation from the discussion, compute and record the circuit impedance magnitude and angle.
13. Compute and record the percent difference between the pairs of values of Z and θ.
14. Repeat steps 5 through 13 for frequencies of 100, 150, 200, 250, 300, 400, 500, and 600 Hz.
15. On a single sheet of graph paper, plot curves of resistance, reactance and impedance magnitudes versus frequency.
16. Replace the inductor with the 1 μF capacitor and repeat steps 5 through 15.
17. Add the inductor in series with the capacitor and repeat steps 5 through 14.
18. On a single sheet of graph paper, plot R, X_L, X_C, and Z versus frequency.

ANALYSIS GUIDE. In the analysis of these data you should carefully consider the manner in which the circuit impedance magnitude and angle were measured and the extent to which these measurements agreed with the calculations. Also, consider the manner in which the impedance varied with frequency.

PROBLEMS

1. What would be the series impedance of a 5KΩ resistor and a 50 H coil at 1.2 Hz?
2. What would be the effective current through a 14 H choke whose winding resistance is 470 ohms if it is connected to a source having an output of 250 sin 754t volts?
3. At what frequency would the coil in problem 2 have the same reactance as an 8 μF capacitor?
4. What would be the series circuit impedance of the coil in problem 2 and the capacitor in problem 3 at the frequency found in problem 3?

experiment 21 PARALLEL IMPEDANCES

INTRODUCTION. As is the case with DC resistances, complex impedances are frequently connected in parallel. In this experiment we shall consider analysis techniques which are appropriate for use with such parallel circuits.

Consider the network shown in figure 21-1.

The impedances, $Z_1 \angle \theta_1$, $Z_2 \angle \theta_2$ and $Z_3 \angle \theta_3$, are all connected in parallel; therefore, we see from Kirchhoff's law that

$$i_T = i_1 + i_2 + i_3$$

From Ohm's law we know that

$$i_T = \frac{e_T}{Z_T \angle \theta_T}, \quad i_1 = \frac{e_T}{Z_1 \angle \theta_1},$$

$$i_2 = \frac{e_T}{Z_2 \angle \theta_2} \text{ and } i_3 = \frac{e_T}{Z_3 \angle \theta_3}$$

where $Z_T \angle \theta_T$ is the total effective impedance of the network. Cancelling e_T in each term renders

$$\frac{1}{Z_T \angle \theta_T} = \frac{1}{Z_1 \angle \theta_1} + \frac{1}{Z_2 \angle \theta_2} + \frac{1}{Z_3 \angle \theta_3} \qquad \textbf{(21.1)}$$

This equation should be recognized as the complex equivalent of the one for resistance in parallel.

Of special interest is the case in which only two parallel impedances are present. In such a case, equation 21.1 becomes

$$\frac{1}{Z_T \angle \theta_T} = \frac{1}{Z_1 \angle \theta_1} + \frac{1}{Z_2 \angle \theta_2}$$

which may be algebraically rearranged into the familiar product-over-sum form

$$Z_T \angle \theta_T = \frac{Z_1 \angle \theta_1 \quad Z_2 \angle \theta_2}{Z_1 \angle \theta_1 + Z_2 \angle \theta_2} \qquad \textbf{(21.2)}$$

It is perhaps worthwhile to work through a numerical example to illustrate how the complex quantities are handled in actual calculations. Let us suppose that the Z_1 branch in figure 21-1 is composed of a 75 mhy coil having a winding resistance of 250 ohms, and that the Z_2 branch is a 0.2 mfd capacitor. Figure 21-2 shows the circuit diagram of this circuit arrangement.

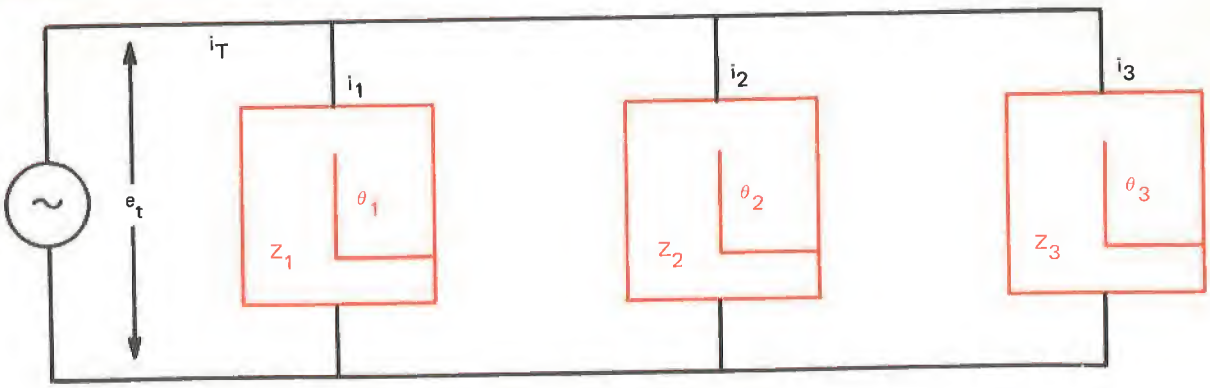


Fig. 21-1 A Complex Parallel Circuit

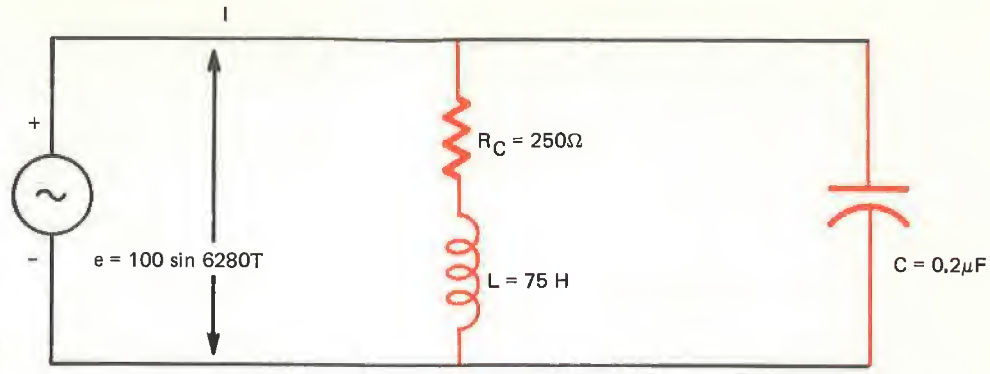


Fig. 21-2 A Typical Complex Parallel Circuit

Finally, let us suppose that we wish to determine the effective current which will flow from the 100 sin 6280T volt source. We may compute the current as follows:

1. Using the source voltage, determine the effective source voltage and the source frequency.

$E = .707\ E_m = .707 \times 100 = 70.7$ volts

$\omega = 2\pi f = 6280$

$f = \frac{6280}{2\pi} = \frac{6280}{6.28} = 1000$ Hz

2. Using f, L and C, compute X_L and X_C.

$X_L = 2\pi f L = 6.28 \times 10^3 \times 75 \times 10^{-3}$

$= 470$ ohms

$X_C = \frac{1}{2\pi f C} = \frac{1}{6.28 \times 10^3 \times 2 \times 10^{-7}}$

$= 795$ ohms

3. Using R_C, X_L and X_C, compute $Z_1 \angle \theta_1$ and $Z_2 \angle \theta_2$.

$Z_1 \angle \theta_1 = R_C + j\ X_L = 250 + j\ 470$

$= 532 \angle 62^\circ$ ohms

$Z_2 \angle \theta_2 = -jX_C = -j\ 795$

$= 795 \angle -90^\circ$ ohms

4. Using $Z_1 \angle \theta_1$ and $Z_2 \angle \theta_2$, determine $Z_T \angle \theta_T$.

$$Z_T \angle \theta_T = \frac{Z_1 \angle \theta_1 \quad Z_2 \angle \theta_2}{Z_1 \angle \theta_1 + Z_2 \angle \theta_2}$$

$$= \frac{532 \angle 62^\circ \times 795 \angle -90^\circ}{250 + j\ 470 - j\ 795}$$

$$Z_T \angle \theta_T = \frac{42.3 \times 10^4 \angle -28^\circ}{250 - j\ 325}$$

$$= \frac{42.3 \times 10^4 \angle -28^\circ}{410 \angle -52^\circ}$$

$$Z_T \angle \theta_T = 1040 \angle 24^\circ \text{ ohms}$$

5. Using E and $Z_T \theta_T$, compute I.

$$I = \frac{E}{Z_T \angle \theta_T} = \frac{70.7 \quad 0^\circ}{1040 \angle 24^\circ}$$

$$= 0.068 \angle -24^\circ \text{ amps}$$

While the computations in this example are somewhat lengthy, the technique is substantially appropriate for use with resistances in parallel. One should notice, however, that due to the phase shift involved, *the total impedance need not be less than the individual branch impedances.*

There is an alternate method which may be employed in analyzing circuits of the type shown in figure 21-1.

If we define the *admittance* (Y) of a circuit to be equal to

$$Y = \frac{1}{Z} \text{ mhos} \qquad (21.3)$$

then equation 21.1 becomes

$$Y_T = Y_1 + Y_2 + Y_3 \text{ mhos} \qquad (21.4)$$

Since the impedance is a complex quantity, then it follows that the admittance is also complex and may, therefore, be written in the form

$$Y \underline{|\theta} = G \pm j\,\beta \text{ mhos} \qquad (21.5)$$

where *G* is the real part of the admittance and is called the *conductance* of the circuit. β, on the other hand, is the imaginary component of the admittance and is termed the *susceptance* of the circuit.

The previously cited example can be solved using circuit admittances as follows:

1. Determine E = 70.7 volts, F = 1000 Hz as before.

2. Determine X_L = 470 ohms, X_C = 795 ohms as before.

3. Determine $Z_1 = 532 \underline{|62^\circ}$ ohms, $Z_2 = 795 \underline{|-90^\circ}$ ohms as before.

4. Determine Y_1 and Y_2.

$$Y_1 = \frac{1}{Z_1} = \frac{1}{532 \underline{|62^\circ}}$$

$$= 0.001875 \underline{|-62^\circ}$$

$$= 0.00088 - j\,0.000165 \text{ mhos}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{795 \underline{|-90^\circ}}$$

$$= 0.00126 \underline{|90^\circ} = +j\,0.00126 \text{ mhos}$$

5. Determine Y_T.

$$Y_T = Y_1 + Y_2 = 0.00088 - j\,0.00165$$

$$+j\,0.000126 = 0.00088 - j\,0.00039$$

$$Y_T = 0.00096 \underline{|-24^\circ} \text{ mhos}$$

6. Determine I.

$$I = EY = 70.7 \underline{|0^\circ} \times 0.00096 \underline{|-24^\circ}$$

$$= 0.068 \underline{|-24^\circ} \text{ amps}$$

While this method is about the same length as as the first method, the actual computations are somewhat simpler.

MATERIALS

1 Audio generator
1 Multimeter
1 Oscilloscope
1 Resistor ≈ the DC resistance of the 1-H inductor used
1 1-H inductor
1 10μF capacitor (oil filled)
1 75-ohm resistor
1 Linear graph paper

PROCEDURE

1. Measure and record the winding resistance (R_C) of the 1-H inductor.
2. Construct the circuit shown in figure 21-3.

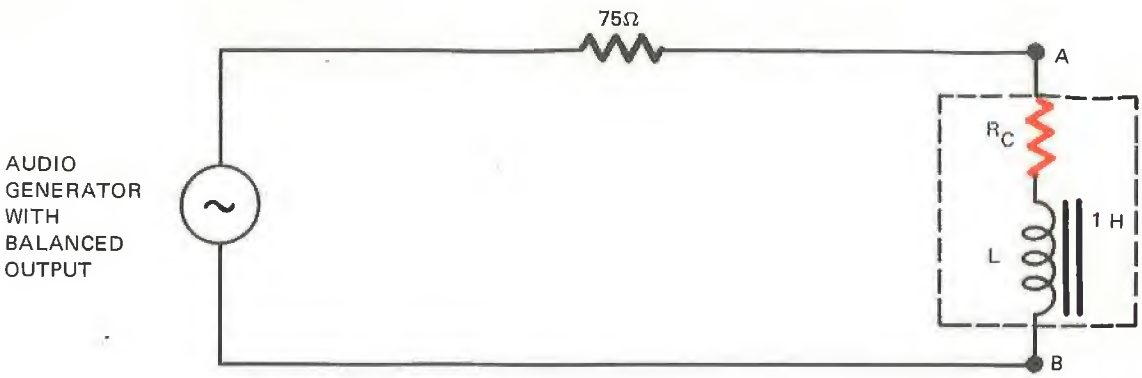


Fig. 21-3 The Experimental Circuit

3. Set the generator for an output frequency of 10 Hz.
4. Set the generator output for an RMS voltage of one-half volt across the 75-ohm resistor.
5. Compute and record the magnitude of the circuit current using the values from step 4.
6. Measure and record the magnitude of the voltage across the inductor.
7. Using the data from steps 5 and 6, determine and record the magnitude of the inductor's impedance (Z_{meas}).
8. Using the oscilloscope, measure and record the phase angle (θ_{meas}) between the voltage across the inductor and the current through it.

Data from Inductor Circuit						R_C =		
f Hz	I	E_L	Z Meas.	θ Meas.	Z Comp.	θ Comp.	% Diff Z	% Diff θ
10								
20								
30								
40								
50								
60								
70								
80								
90								
100								

Fig. 21-4 The Data Table

Data from the RC Circuit								
f Hz	I	E	Z Meas.	θ Meas.	Z Comp.	θ Comp.	% Diff Z	% Diff θ
10								
20								
30								
40								
50								
60								
70								
80								
90								
100								

Data from the Parallel Combination								
f Hz	I	E	Z Meas.	θ Meas.	Z Comp.	θ Comp.	% Diff Z	% Diff θ
10								
20								
30								
40								
50								
60								
70								
80								
90								
100								

Fig. 21-4 The Data Tables (continued)

9. Using the component values and f, compute and record the magnitude (Z_{comp}) and phase angle (θ_{comp}) of the circuit. (Omit the 75-ohm resistor).
10. Compute and record the percent difference between the pairs of values of Z and θ.
11. Repeat steps 4 through 10 for generator frequencies of 20, 30, 40, 50, 60, 70, 80, 90, and 100 Hz.
12. Remove the inductor from the experimental circuit and replace it with the inductor's resistance and the 10-μF capacitor in series between points A and B.
13. Repeat steps 3 through 11 using the inductor's resistance capacitor instead of the inductor.
14. Connect the inductor to points A and B (in parallel with the inductor's resistance and the 10-μF capacitor).
15. Repeat steps 3 through 11 for the parallel combination of the inductor and the RC circuit.
16. On a single sheet of graph paper plot curves of the impedance magnitude of the inductor, RC circuit, and parallel combination versus frequency.

ANALYSIS GUIDE. In analyzing these data consider each of the following points:

a) What effect did the 75-ohm resistor have on the results?
b) Does your plot of the impedance agree with the plots of the individual branches?
c) Were the phase angle measurements as accurate as the impedance magnitude measurements?
d) How could the circuit admittances have been measured? Would the experimental results be the same if admittances were used instead of impedances?

PROBLEMS

1. Compute the circuit admittance for each of the three experimental circuits at frequencies of 10, 50, and 100 Hz.
2. Use the admittances from problem 1 and equation 21.3 to compute the experimental circuit impedances at 10, 50, and 100 Hz.
3. How do the values in problem 2 compare with the values measured in the experiment?
4. On the experimental plot of impedance magnitude vs frequency, there should be a frequency where $|Z_{RC}| = |Z_L|$. Also at this resonant frequency, the plot of the parallel combination of the impedance should be a maximum greater than the magnitude of either individual impedance.

   If these impedances were purely resistive the parallel combination of this frequency would be equal to one-half the value of either resistance since they are equal.

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 R_1}{R_1 + R_1} = \frac{R_1 R_1}{2R_1} = \frac{R_1}{2}$$

   Since the winding resistance of the inductor used is approximately equal to the extra resistor, explain why the value of the plot of the parallel impedances does not agree with the value of purely resistive elements.

experiment 22 AC POWER

INTRODUCTION. Due to the *phase shift* between the voltage and current that may occur in an AC circuit, the power applied to the circuit may be less than expected. In this experiment we shall examine the power relations in a complex AC circuit.

DISCUSSION. If an AC source is connected to a circuit containing only resistances, the circuit current will be in phase with the voltage. The instantaneous power dissipated by the resistance will be

$$p = ei \qquad (22.1)$$

If we sketch the voltage, current, and power, the result would be as indicated in figure 22-1. In such a case (resistive loads), the average power dissipated by the load is

$$P = EI = E^2/R = I^2R = 1/2\ Pm \qquad (22.2)$$

where E and I are effective (RMS) values and Pm is the maximum instantaneous power.

If the load attached to the AC source is *purely reactive* (either capacitive or inductive), the conditions mentioned above *do not hold.* Consider, for example, the circuit shown in figure 22-2.

In this circuit the current and voltage will be 90° apart, as indicated by figure 22-3. The instantaneous power will still be

$$p = ei$$

and therefore, as shown in figure 22-3, will be positive when e *and* i are *both* positive or negative (first and third quadrants).

The value of p will be negative when *either* e *or* i are negative (second and fourth quadrants). The average power taken over one complete cycle will be equal to zero.

If we measure the effective voltage and current in a purely reactive circuit, their product will, in general, *not be equal to zero.* We call this product the *reactive (or imaginary) power* supplied to the circuit, and we express it in units called VARs (for volt-amps-reactive).

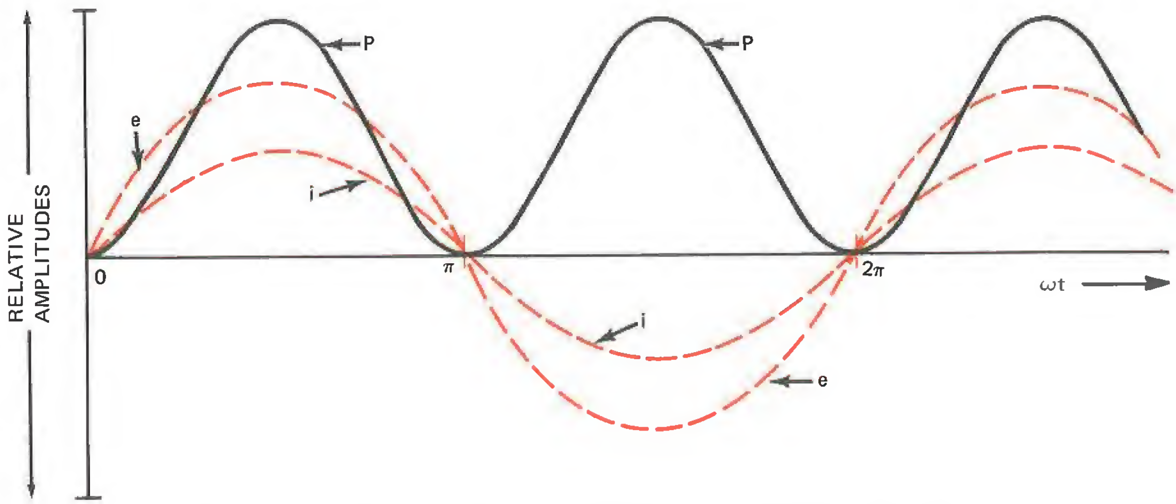


Fig. 22-1 Voltage, Current, and Power in a Resistive Circuit

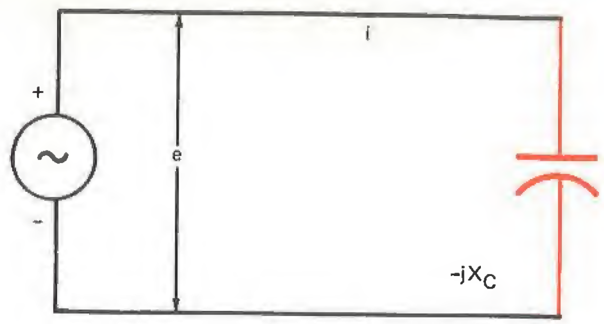


Fig. 22-2 A Purely Capacitive Load

In a circuit containing both reactance and resistance, the phase angle between the voltage and current will have some value between 0° and ±90°. Figure 22-4 shows a sketch of the voltage, current, and power in such a case. The instantaneous power in this case is still related to the instantaneous voltage and current by p = ei.

In this case, the average power taken over one cycle will be positive; it will not be as great as in the purely resistive case. In this instance, we have three separate quantities to consider:

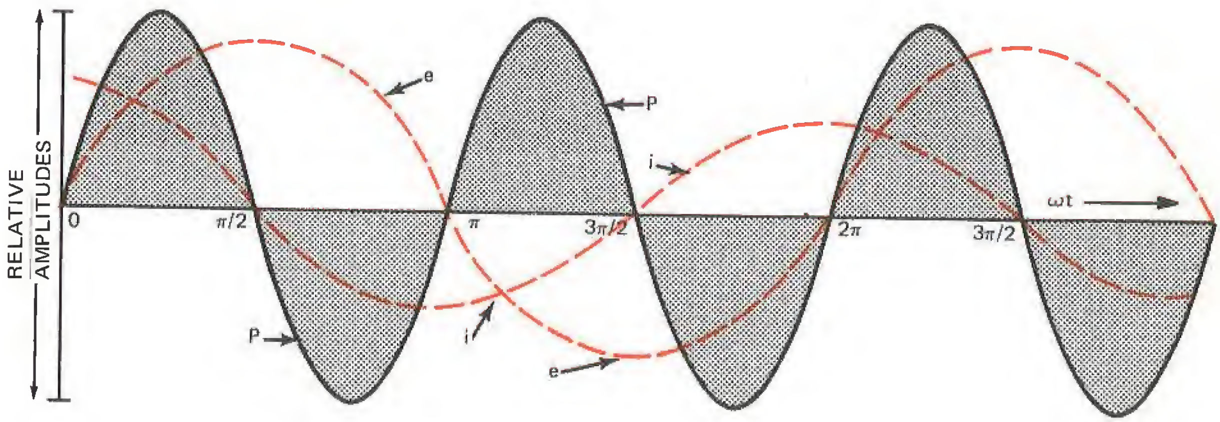


Fig. 22-3 Voltage, Current, and Power in a Purely Reactive Circuit

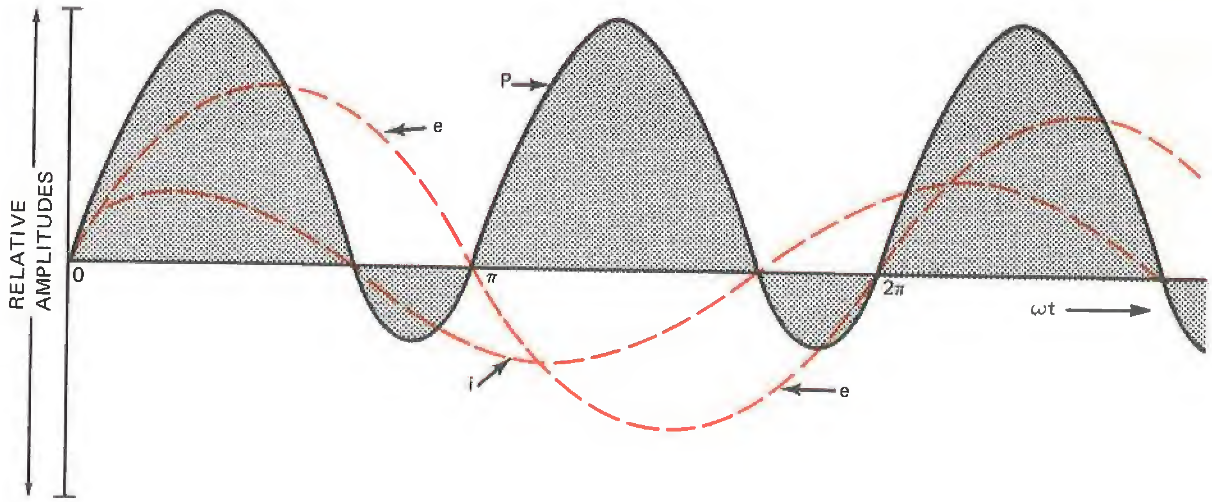


Fig. 22-4 Voltage, Current, and Power in a Complex Circuit

a. The *Apparent Power* (P_{app}), which is the product of the RMS voltage and current,

$$P_{app} = E_T I_T \text{ (in volt-amps)} \tag{22.3}$$

b. The *Average Power* (P_{ave}), which is the power loss in the circuit resistance,

$$P_{ave} = E_R I_T \text{ (in watts)} \tag{22.4}$$

c. The *Reactive Power* (P_X), which is the imaginary power supplied to the circuit,

$$P_X = E_X I_T \text{ (in VARs)} \tag{22.5}$$

Let us now consider a circuit in which this last set of conditions is appropriate. Figure 22-5 shows one such circuit.

The total impedance of the circuit will be

$$Z = R + j X_L$$

If we multiply each term by the value of the circuit current, the result is

$$I_T Z = I_T R + j I_T X_L$$

However, we observe that

$$I_T Z = E_T, \quad I_T R = E_R \quad \text{and} \quad I_T X_L = E_X$$

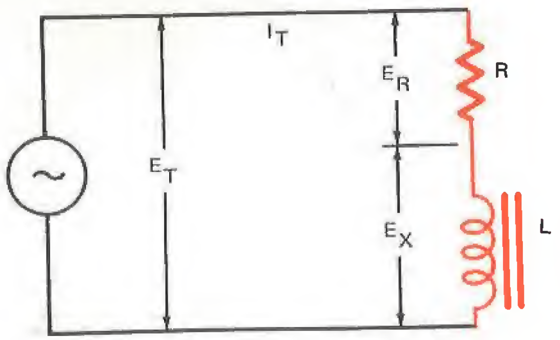


Fig. 22-5 A Complex Circuit

We may therefore write

$$E_T = E_R + j E_X \tag{22.6}$$

Now, if we again multiply each term by I_T we have

$$E_T I_T = E_R I_T + j E_X I_T$$

And since these are the three power-related quantities discussed above, we may write

$$P_{app} = P_{ave} + j P_X \tag{22.7}$$

This relationship may be expressed in the form of a right triangle as shown in figure 22-6, which is called *"the power triangle."*

Also, we can express the average power as

$$P_{ave} = P_{app} \cos\theta \tag{22.8}$$

or

$$P_{ave} = E_T I_T \cos\theta \tag{22.9}$$

where θ is the phase angle between E_T and I_T.

In some cases the cosine of θ is called the power factor (pf) of the circuit; in which cases, we may write

$$pf = \cos\theta \tag{22.10}$$

and

$$P_{ave} = E_T I_T \text{ (pf)} \tag{22.11}$$

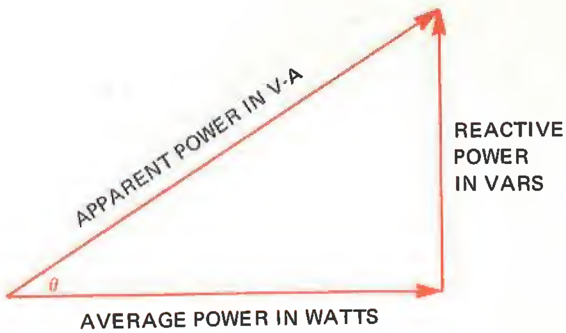


Fig. 22-6 The Power Triangle

In the case of a DC circuit or a purely resistive AC circuit, the value of θ is zero. Therefore, pf = cos θ = 1. In the case of a purely reactive AC circuit, $\theta = 90^\circ$ and pf = cos θ = 0. As an overall result, then, we observe that equation 22.11 holds for all cases.

MATERIALS

1 Oscilloscope

1 Multimeter

1 AC ammeter

1 Wattmeter

1 Variable transformer

1 10-μF capacitor

1 250-ohm resistor (20 watt)

PROCEDURE

1. Assemble the circuit shown in figure 22-7.
2. Set the variable transformer for a total circuit voltage of about 100 volts RMS.
3. Measure and record the values of:
   a. The circuit voltage (E_T)
   b. The circuit current (I_T)
   c. The voltage across the resistor (E_R)
   d. The voltage across the capacitor (E_C)
4. Replace the ammeter with the current coil of the wattmeter.

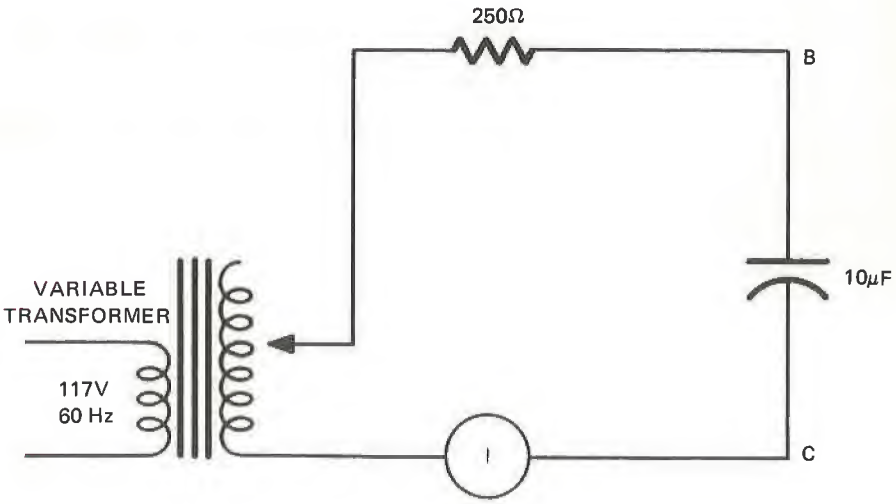


Fig. 22-7 The Experimental Circuit

Quantity	Value
E_T	
I_T	
E_R	
E_C	
P_{ave}	
P_R	
P_C	
θ	
P_{ave}	
P_{app}	
P_X	
pf	
% Diff P_{ave}	
I_T Comp.	
% Diff I_T	
P_{app} Comp.	
% Diff P_{app}	

Fig. 22-8 The Data Table

5. Connect the common side of the wattmeter voltage coil to point C and connect the remaining voltage coil lead to measure:
   a. The voltage across both the resistor *and* the capacitor. Record the average power supplied to the circuit (P_{ave}).
   b. The capacitor voltage only and record the average power supplied to the capacitor (P_C).
6. Move the wattmeter voltage coil leads to the points necessary to measure the voltage across the resistor. Record the power dissipated by the resistor (P_R).
7. Using the oscilloscope, measure and record the phase angle between the circuit voltage and current (θ).
8. Using the circuit current and E_T, E_R, or E_C, as appropriate, compute and record P_{ave}, P_{app}, and P_X.
9. Compute and record the circuit power factor (pf) using θ measured in step 7.
10. Compute the percent difference between the two values of P_{ave}.
11. Compute the value of X_C using C and F.
12. Compute Z_T using $Z_T = R - jX_C$.
13. Compute I_T using $I_T = E_T/Z_T$. Record I_T in the data table (I_T comp).
14. Compute the percent difference in I_T and I_T comp.
15. Compute the value of P_{app} using E_T and I_T comp. (P_{app} comp).
16. Compute the percent difference between P_{app} and P_{app} comp.

ANALYSIS GUIDE. In the analysis of these data, consider carefully the difference between P_{ave}, P_{app}, and P_X. How do the results compare to the relationships specified in the discussion?

PROBLEMS

1. In a given circuit the average power is measured and found to be 47.3 watts. If the circuit voltage and current are 117 volts and 1.05 amps respectively, what is the magnitude of:
   a. The power factor?
   b. The circuit reactance?
   c. The phase angle between the circuit voltage and current?

2. In a complex circuit, why is the apparent power always larger than the average power?

3. A certain circuit containing an inductor and a resistor is found to have a power factor of 0.630. How can the power factor be changed to 1.00 if the circuit resistance is 12.9K and the input frequency is 1000 Hz?

experiment 23 SERIES RESONANCE

INTRODUCTION. When a resistor, capacitor, and inductor are connected in series, a frequency may be found at which the reactive components of the impedance cancel each other. In this experiment we shall examine the circuit performance under such conditions.

DISCUSSION. Let us consider the circuit shown in figure 23-1. The magnitudes of the two reactances in the circuit will be determined by

$$X_L = 2\pi fL \quad \text{and} \quad X_C = \frac{1}{2\pi fC}.$$

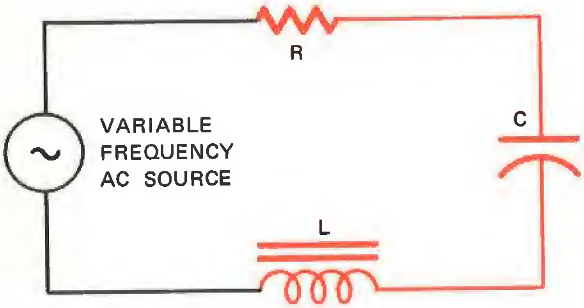


Fig. 23-1 A Series RLC Circuit.

If we plot the reactances versus frequency, the result will be as shown in figure 23-2. A plot of the resistance and circuit impedance is also shown. From this sketch we should take note of the fact that since the circuit impedance is given by

$$Z = R + j\,(X_L - X_C),$$

there is a point (f_o) at which

$$X_L = X_C \tag{23.1}$$

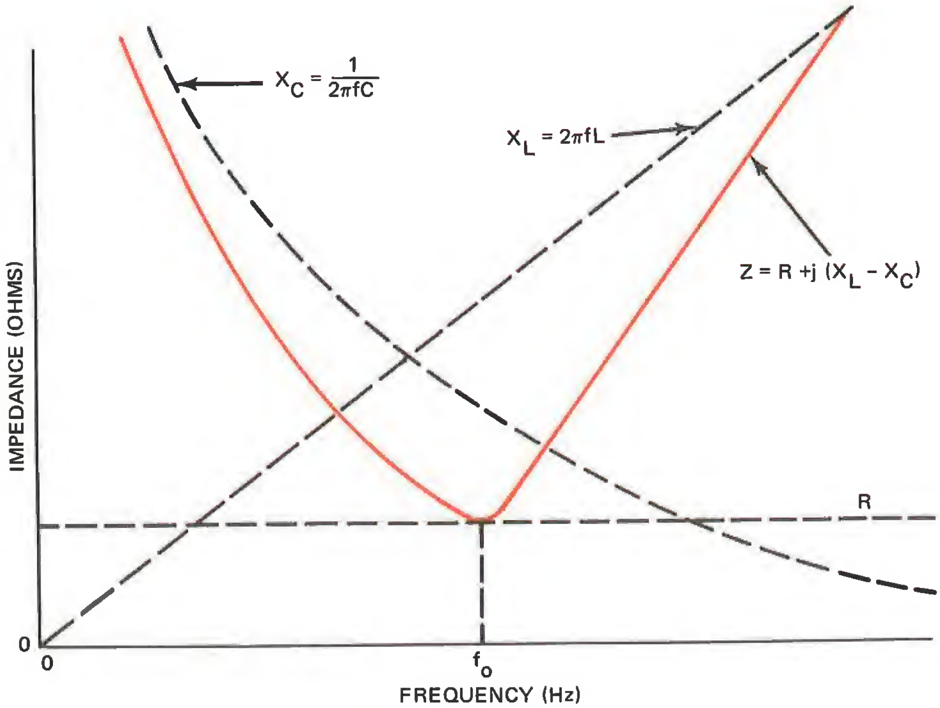


Fig. 23-2 Impedance of Series RLC Circuit

and therefore

$$Z = R \tag{23.2}$$

at this point.

We can determine the point at which Z equals R by returning to equation 23.1,

$$X_L = X_C \quad \text{at} \quad f_o.$$

Therefore, we may write

$$2\pi f_o L = \frac{1}{2\pi f_o C}.$$

Dividing each side by $2\pi L$ renders

$$f_o = \frac{1}{4\pi^2 f_o LC}$$

and multiplying by f_o gives us

$$f_o^2 = \frac{1}{4\pi^2 LC}.$$

Finally, taking the square root of each side provides

$$\boxed{f_o = \frac{1}{2\pi\sqrt{LC}}} \tag{23.3}$$

which is the frequency at which $X_L = X_C$ and $Z = R$. This frequency is called the *series resonant frequency* of the circuit.

If we apply a constant level AC voltage to the circuit at the resonant frequency, the circuit current will be

$$I_T = \frac{E_T}{Z_T} = \frac{E_T}{R} \tag{23.4}$$

The value of the voltage drop across the resistor will be the product of the circuit current and the ohmic value of the resistor,

$$E_R = I_T R$$

But since $I_T = E_T/R$, the resistor voltage becomes

$$E_R = E_T \tag{23.5}$$

In other words, the voltage across the resistor is equal to the applied voltage. Similarly, the voltage across the inductance is

$$E_L = I_T X_L$$

and since $I_T = E_T/R$, we have

$$E_L = E_T \frac{X_L}{R} \tag{23.6}$$

The ratio of X_L to R at resonance is frequently called the *quality factor* (Q_o) of the circuit. That is

$$\boxed{Q_o = \frac{X_L}{R} \text{ at resonance}} \tag{23.7}$$

The equation for the inductive voltage may, therefore, be rewritten as

$$E_L = Q_o E_T \tag{23.8}$$

In like manner, the capacitive voltage is

$$E_C = I_T X_C = E_T \frac{X_C}{R}$$

and since $X_C = X_L$ at resonance, we may write

$$E_C = Q_o E_T \tag{23.9}$$

In a practical case, the value of X_L will usually be from two to over a hundred times the value of R. Typical values of Q_o will therefore range from about two to over a hundred. *The voltage across the inductor or capacitor may therefore be many times greater than the applied voltage.* This phenomena is referred to as the *resonant rise in voltage* and occurs only at or near the resonant frequency.

In general, the circuit current for a fixed applied voltage is

$$I_T = \frac{E_T}{Z}$$

In other words, I_T varies inversely as does the circuit impedance. Figure 23-3 shows a sketch of circuit impedance and current versus frequency.

The change in frequency $(f_2 - f_1)$ between the points where current is down to 70.7% of its highest value is called the *bandwidth* (BW) of the circuit:

$$BW = f_2 - f_1 \qquad \textbf{(23.10)}$$

We may determine the size of the bandwidth by observing that at f_2, $I_T = 0.707\ E_T/R$. From Ohm's law we know that

$$Z_T = \frac{E_T}{I_T}$$

Therefore, at f_2 we have

$$Z_T = \frac{E_T}{I} = \frac{E_T}{0.707\ E_T/R} = 1.414R = \sqrt{2}\ R$$

Since the impedance magnitude is given by

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

we see that at f_2,

$$\sqrt{2}\ R = \sqrt{R^2 + (X_L - X_C)^2}$$

Squaring both sides gives

$$2R^2 = R^2 + (X_L - X_C)^2.$$

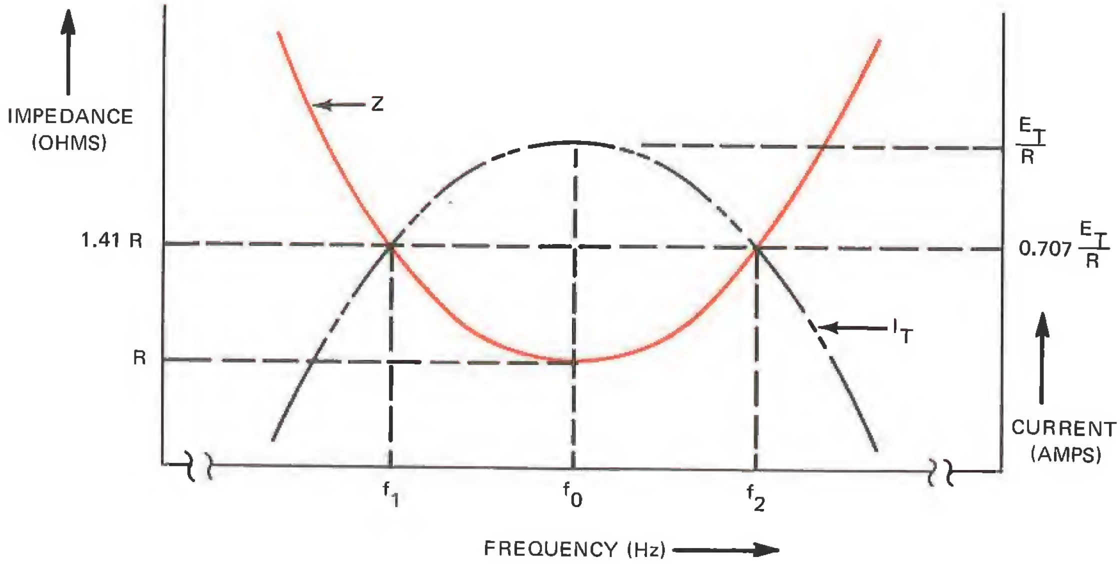


Fig. 23-3 Impedance and Current Versus Frequency

Subtracting R^2 from each side,

$$R^2 = (X_L - X_C)^2$$

or

$$R = X_L - X_C$$

Because X_L is directly proportional to frequency and X_C is inversely proportional, X_L and X_C must change by approximately the same amount. That is, X_L must increase by about 1/2R while X_C decreases by about 1/2R. At f_2, then, the total inductive reactance will be

$$2\pi f_2 L = 2\pi f_o L + 1/2R$$

Solving for f_2 renders

$$f_2 = f_o + \frac{R}{4\pi L}$$

Using the same reasoning for f_1 (X_L decreases by 1/2R while X_C increases by the same amount) gives us

$$f_1 = f_o - \frac{R}{4\pi L}$$

The bandwidth of the circuit has been defined as

$$BW = f_2 - f_1.$$

Therefore,

$$BW = f_o + \frac{R}{4\pi L} - f_o + \frac{R}{4\pi L}$$

or finally,

$$\boxed{BW \approx \frac{R}{2\pi L}} \qquad \textbf{(23.11)}$$

If we divide both sides by f_o, we will have

$$\frac{BW}{f_o} = \frac{R}{2\pi f_o L}$$

The righthand ratio should be recognized as $1/Q_o$; therefore

$$\frac{BW}{f_o} = \frac{1}{Q_o}$$

or

$$\boxed{BW = \frac{f_o}{Q_o}} \qquad \textbf{(23.12)}$$

This relationship is very handy in many practical cases.

If the AC source in figure 23-1 is replaced by a DC source and a switch, as shown in figure 23-4, another property of resonant circuits may be observed. *The resistor (R_s) is included only to prevent the DC supply from being overloaded when the switch is closed.*

Let us presume that the switch is open and the capacitor is charged to the supply potential. When we close the switch, current flows through R and L, establishing a magnetic field around the inductor. When the charge on C diminishes, the field around L starts to collapse, tending to charge C in the opposite direction. When the field is completely collapsed, C is partially charged with reversed polarity, and the process starts again in the opposite direction. The energy in the circuit continues to alternately charge C and L until it is eventually all dissipated by the circuit resistance.

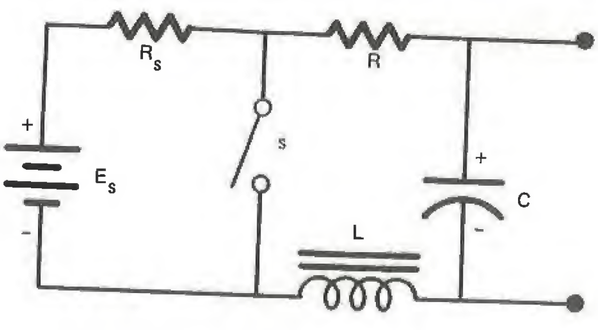


Fig. 23-4 An RLC Ringing Circuit

If we observe the capacitor voltage with an oscilloscope, it will appear as shown in figure 23-5. This resonant phenomena is termed *ringing* in the resonant circuit. The frequency of the ringing will be the same as the resonant frequency of the circuit.

The ringing frequency (and therefore the resonant frequency) may be determined by measuring the period as the circuit rings.

Substantially the same thing happens when the switch is opened charging the circuit.

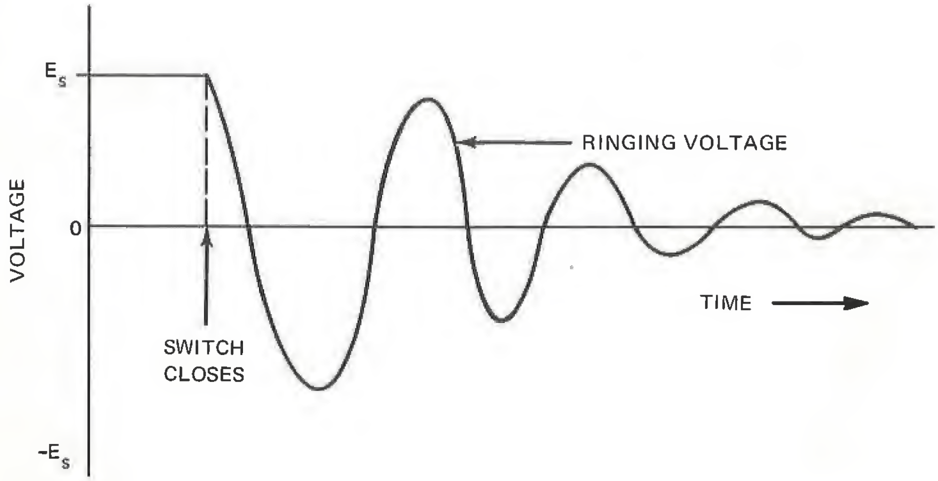


Fig. 23-5 Ringing in an RLC Circuit

MATERIALS

- 1 Audio generator
- 1 Oscilloscope
- 1 Multimeter
- 1 Variable DC supply
- 1 1-H inductor
- 1 1-μF capacitor
- 1 SPST switch
- 1 100-ohm resistor
- 2 Sheets of linear graph paper

PROCEDURE

1. Measure and record the value of the coil's winding resistance (R_C).
2. Assemble the circuit shown in figure 23-6.
3. Connect the VOM to measure the generator output.
4. Set the generator for maximum output at 100 Hz. Slowly vary the generator frequency between 100 and 200 Hz. Locate the frequency at which the generator has the *least* output voltage. Make a mental note of the value of the output voltage at this point.

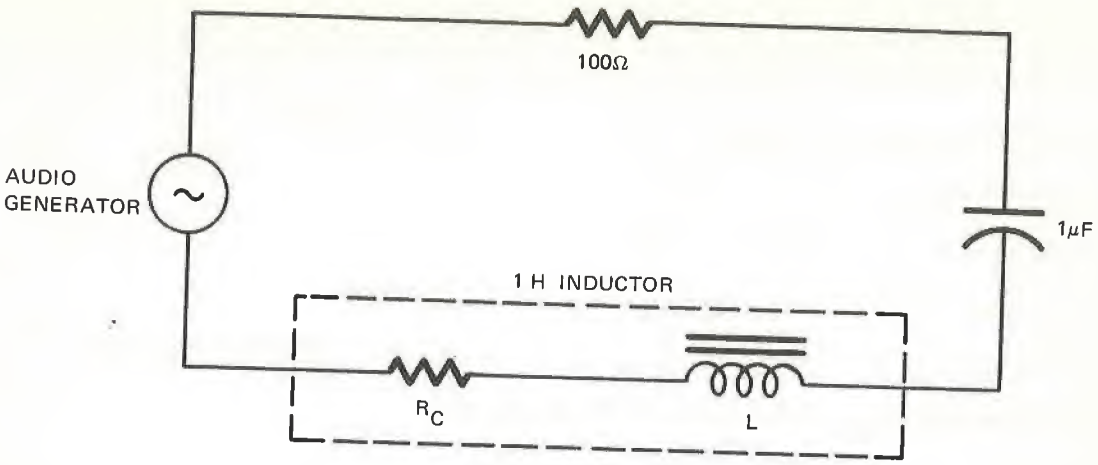


Fig. 23-6 The Initial Experimental Circuit

5. Choose some convenient value of generator voltage that is less than the value noted above. Record this chosen value as E_T and maintain the generator output at this level throughout the remainder of the experiment.
6. Remove the VTVM and connect the oscilloscope to monitor the generator output. **Hold the generator output voltage constant during the following steps!**
7. Set the generator frequency to 50 Hz and connect the VTVM to measure the voltage across the 100-ohm resistor.
8. Record the voltage across the resistor (E_1) for generator frequencies of: 50, 60, 70, 80, 90, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150, 155, 160, 165, 170, 175, 180, 185, 190, 195, 200, 210, 220, 230, 240, 250, and 260 Hz.
9. Using Ohm's law, compute and record the circuit current, I_T, and impedance, Z_T, at each frequency in step 8.
10. On a single sheet of graph paper, plot both I_T and Z_T versus frequency.
11. Set the generator to the frequency at which E_1 is the greatest. Record this as f_o (meas).
12. Measure the voltage across the capacitor (E_C).
13. Measure the voltage across the 100-ohm resistor and compute the maximum circuit current, I_o.
14. Determine the total circuit resistance, R_T.
15. Using the data from steps 13 and 14, compute the voltage across the total circuit resistance (E_R).
16. Using the measured value of f_o, compute the value of the inductive reactance (X_L).
17. Compute and record the circuit, Q_o.

18. Using Q and E_T, compute the value of E_C.
19. Compute the percent difference between E_T and E_R.
20. Compute the percent difference between the two values of E_C.
21. Using the values of L and C, compute f_o.
22. Compute the percent difference between the two values of f_o.
23. From your curve, determine the circuit bandwidth (BW meas).
24. Using data from steps 11 and 17, compute the value of the bandwidth (BW comp).
25. Compute the percent difference between the two values of BW.
26. Assemble the circuit shown in figure 23-7.
27. Open the switch S and set the variable DC supply for 15 volts output.
28. Connect the oscilloscope for viewing the voltage across the capacitor.
29. Snap the switch closed and view the waveform of the voltage across the capacitor.
30. On a sheet of graph paper make an accurate sketch of the capacitor voltage waveform.

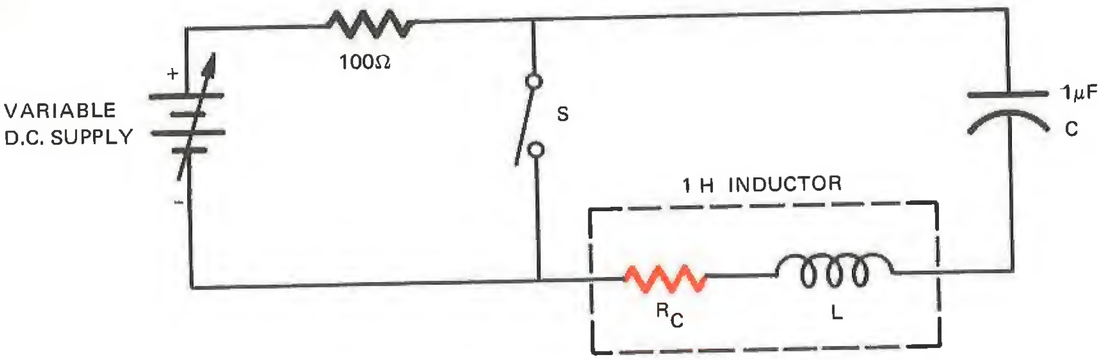


Fig. 23-7 The Second Experimental Circuit

ANALYSIS GUIDE. In analyzing these data, you should examine each of the numbered equations in the discussion and consider the extent to which your data confirmed them. What practical applications can you think of that would employ resonant circuits?

PROBLEMS

1. What is the resonant frequency of the circuit shown in figure 23-9?
2. What is the circuit Q_o in figure 23-9?
3. Based on the equation given in the discussion, what would be the bandwidth of this circuit?
4. What is the power factor of a circuit at resonance?

Qty	Value
R_C	
E_T	
f_o meas.	
E_C meas.	
I_O	
R_T	
E_R	
X_L	
Q_o	
E_C comp.	
% Diff E_R, E_T	
% Diff E_C	
f_o comp.	
% Diff f_o	
BW meas.	
BW comp.	
% Diff BW	

f (Hz)	E_1	I_T	Z_T	f (Hz)	E_1	I_T	Z_T
50				155			
60				160			
70				165			
80				170			
90				175			
100				180			
105				185			
110				190			
115				195			
120				200			
125				210			
130				220			
135				230			
140				240			
145				250			
150				260			

Fig. 23-8 The Data Tables

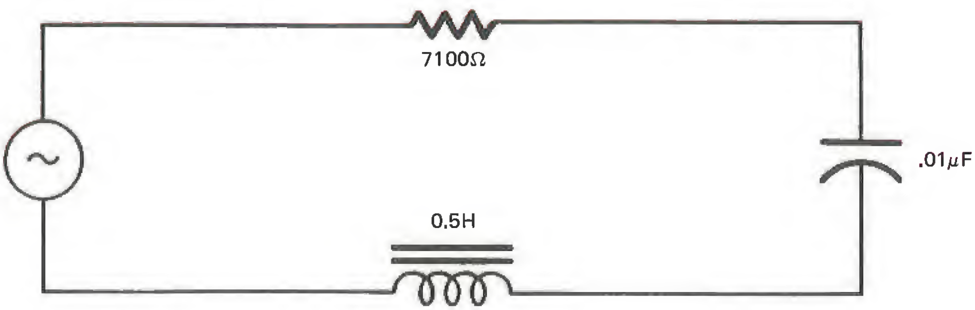


Fig. 23-9 Circuit for Problems 1, 2, and 3.

experiment 24 PARALLEL RESONANCE

INTRODUCTION. Parallel RLC circuits exhibit a number of characteristics which are similar to those of series resonant circuits. In this experiment we shall examine some of these characteristics.

DISCUSSION. Let us direct our attention to the parallel circuit shown in figure 24-1.

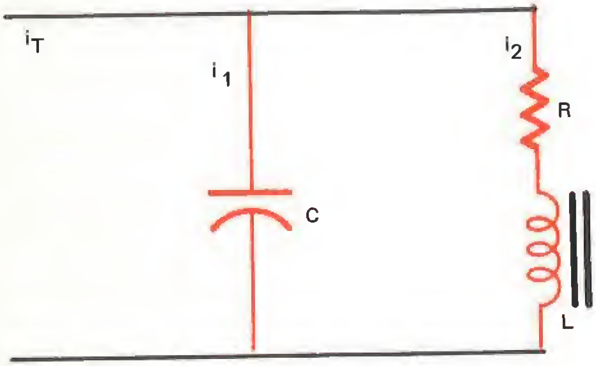


Fig. 24-1 A Parallel RLC Circuit

If we apply a variable frequency constant voltage across the circuit terminals, the current in the capacitive branch will be

$$i_1 = \frac{e}{-jX_c} = +j2\pi fC(e)$$

From this equation we observe that the capacitive current is directly related to the frequency.

On the other hand, the current in the inductive branch will be

$$i_2 = \frac{e}{Z_2} = \frac{e}{R + jX_L} = \frac{e(R - jX_L)}{|Z_2|^2}$$

$$= \frac{eR}{|Z_2|^2} - j\frac{eX_L}{|Z_2|^2}$$

The total circuit current will be the sum of these two branch currents

$$i_T = i_1 + i_2 = \frac{eR}{|Z_2|^2} + j\left(\frac{e}{X_c} - \frac{eX_L}{|Z_2|^2}\right) \qquad \textbf{(24.1)}$$

A plot of the three current components would appear as shown in figure 24-2.

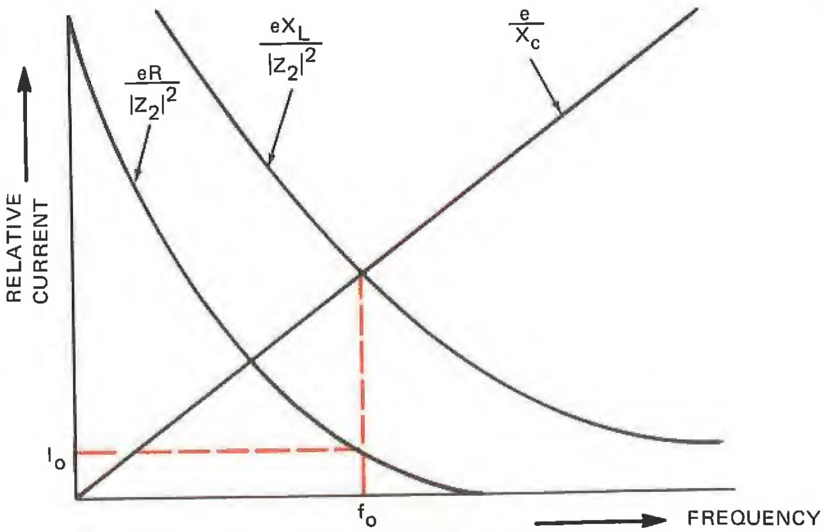


Fig. 24-2 Current versus Frequency

It is apparent that there is a point at which e/X_c is equal to $eX_L/|Z_2|^2$ and at this point the circuit current is

$$I_o = \frac{eR}{|Z_2|^2} \qquad \textbf{(24.2)}$$

This is the point at which the parallel circuit is said to be at resonance. We can determine the frequency at which *resonance* occurs by equating the two reactive current terms

$$\frac{e}{X_c} = \frac{eX_L}{|Z_2|^2}$$

or

$$\frac{1}{X_c} = \frac{X_L}{|Z_2|^2}$$

Substituting $X_c = 1/(2\pi f_o C)$, $X_L = 2\pi f_o L$, and $|Z_2|^2 = R^2 + (2\pi f_o L)^2$ renders

$$2\pi f_o C = \frac{2\pi f_o L}{R^2 + 4\pi^2 f_o^2 L^2}$$

Canceling $2\pi f_o$ on each side gives

$$C = \frac{L}{R^2 + 4\pi^2 f_o^2 L^2}$$

from which we have

$$\frac{L}{C} = R^2 + 4\pi^2 f_o^2 L^2 \qquad \textbf{(24.3)}$$

Solving this relationship for f_o^2 renders

$$f_o^2 = \frac{L - R^2C}{4\pi^2 L^2 C}$$

Dividing both the numerator and denominator by L and regrouping gives us

$$f_o^2 = \frac{1}{4\pi^2 LC}\left(1 - \frac{R^2C}{L}\right)$$

Taking the square root of each side provides

$$f_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R^2C}{L}} \qquad \textbf{(24.4)}$$

From this equation we should notice that the resonant frequency of the parallel circuit differs from that of a series circuit employing the same components by the factor

$$\sqrt{1 - \frac{R^2C}{L}}.$$

If we return to equation 24.3

$$\frac{L}{C} = R^2 + 4\pi^2 f_o^2 L^2$$

and rewrite it, using $X_L^2 = 4\pi^2 f_o^2 L^2$, we have

$$\frac{L}{C} = R^2 + X_L^2$$

Dividing both sides by R^2 gives us

$$\frac{L}{R^2C} = 1 + \frac{X_L^2}{R^2}$$

and we recognize X_L^2/R^2 as Q^2; therefore,

$$\frac{L}{R^2C} = 1 + Q^2$$

or

$$\frac{R^2C}{L} = \frac{1}{1 + Q^2}$$

The equation for resonant frequency (Eq. 24.4) may therefore be rewritten in the form

$$f_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{1}{1+Q^2}}$$

which, if we use the common denominator $(1 + Q^2)$ under the radical on the right, becomes

$$f_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{1+Q^2}} \qquad \textbf{(24.5)}$$

From this equation we notice that if Q is considerably greater than unity, then $1 + Q^2 \approx Q^2$ and $Q^2/(1 + Q^2) \approx 1$. Then

$$f_o' \approx \frac{1}{2\pi\sqrt{LC}} \qquad \textbf{(24.6)}$$

becomes a good approximation of the resonate frequency. In practice this equation is used when Q is about 10 or more. Returning to the equation for circuit current (Eq. 24.1), we observe that a plot of total current versus frequency will be somewhat as shown in figure 24-3.

The total circuit impedance is given by Ohm's law as

$$Z_T = \frac{e_T}{i_T}$$

and is also shown in figure 24-3.

We should observe that the resonant frequency (f_o) given by equation 24.5 and that (f_o') given by equation 24.6 are not exactly the

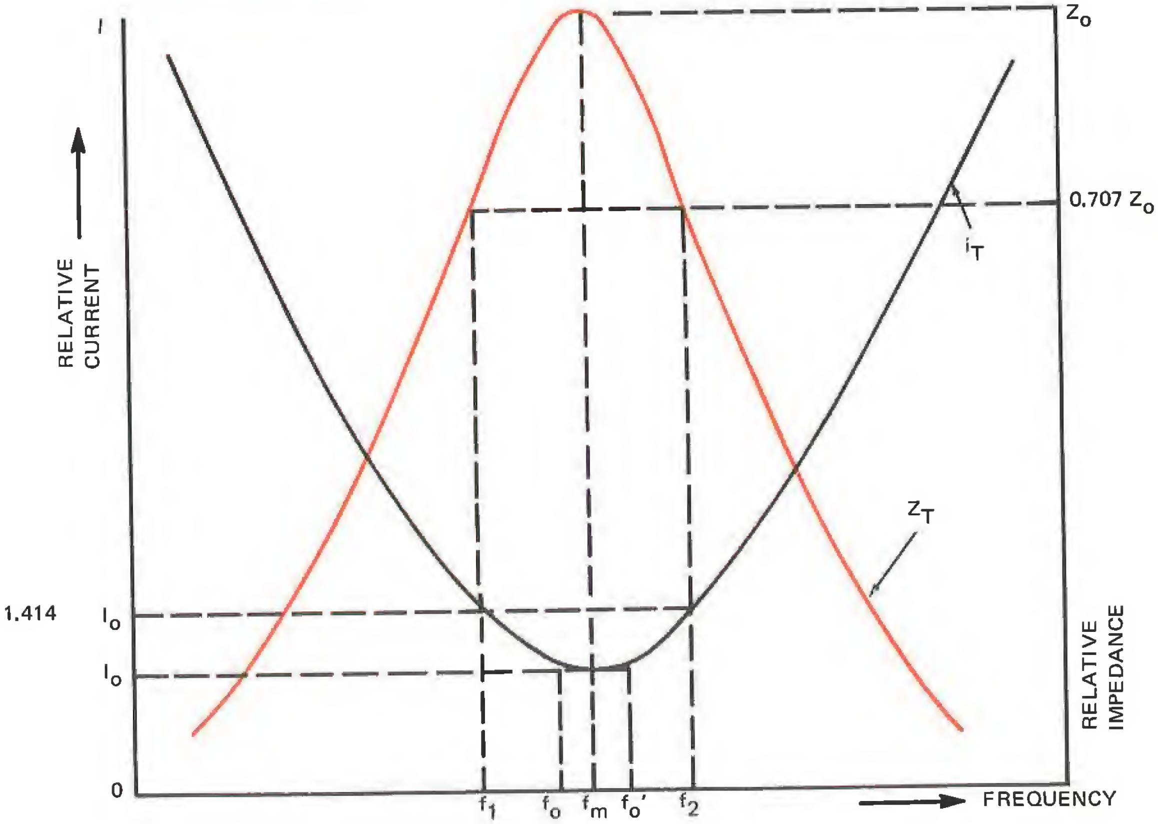


Fig. 24-3 Total Current and Impedance Versus Frequency

same. The frequency at which the total circuit current is at its minimum value (f_m) is also different from f_o and will always be between f_o and f_o'. The derivation of the equation for f_m is beyond the scope of this discussion but can be shown to be

$$f_m = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{1}{4Q^2}} \qquad \textbf{(24.7)}$$

The important thing to notice in this equation is that if $Q = 1/2$ ($Q^2 = 1/4$) then f_m becomes imaginary. This condition is termed *critical damping.* When Q is 1/2 or less, the resonant circuit will not ring when it is subjected to a sudden change in input energy.

We should recall at this point that the bandwidth of a series resonant circuit is

$$BW = \frac{f_o}{Q} \qquad \textbf{(24.8)}$$

In the case of a parallel resonant circuit, this relationship is only approximate but is reasonably accurate for circuits in which the Q is large compared to unity. Equation 24.8 is normally used in cases where the circuit Q is about 10 or above.

Let us once again return to the equation for total circuit current:

$$i_T = \frac{eR}{|Z_2|^2} + j\left(\frac{e}{X_c} - \frac{eX_L}{|Z_2|^2}\right) \qquad \textbf{(24.1)}$$

If we substitute e/Z_T for i_T, we may write

$$\frac{e}{Z_T} = \frac{eR}{|Z_2|^2} + j\frac{e}{X_c} - j\frac{eX_L}{|Z_2|^2}$$

Consideration of this equation suggests that we can represent the original parallel circuit by a three-branch circuit of the type shown in figure 24-4.

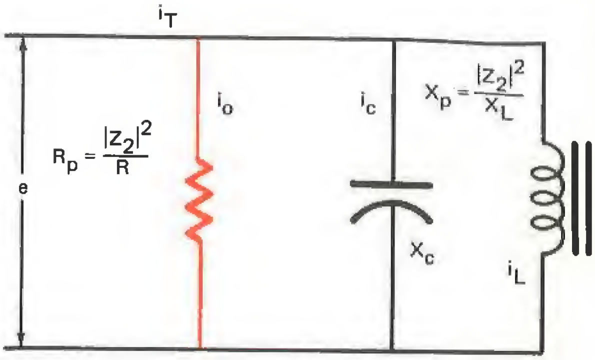


Fig. 24-4 Equivalent Circuit of a Practical Parallel LC Circuit

The Q of the original coil was defined as

$$Q = \frac{X_L}{R} \qquad \textbf{(24.9)}$$

If we use the quantities given in figure 24-4 and solve for R (in terms of R_p and Z_2) and X_L (in terms of X_p and Z_2), the results are

$$X_L = \frac{|Z_2|^2}{X_p} \quad \text{and} \quad R = \frac{|Z_2|^2}{R_p}$$

Substituting these values into equation 24.9 renders

$$Q = \frac{X_L}{R} = \frac{|Z_2|^2/X_p}{|Z_2|^2/R_p} = \frac{R_p}{X_p}$$

At the resonant frequency, i_c must equal i_L and therefore $X_c = X_p$ must also be satisfied. We may therefore write

$$Q = \frac{X_L}{R} = \frac{R_p}{X_p} = \frac{R_p}{X_c} \qquad \textbf{(24.10)}$$

Also, since at resonance, $X_c = X_p$, then total circuit impedance will be

$$Z_o = \frac{e}{i_T} = \frac{|Z_2|^2}{R} = R_p \qquad \textbf{(24.11)}$$

Similarly, we see that

$$Z_o = \frac{|Z_2|^2}{R} = \frac{R^2 + X_L{}^2}{R} = R + \frac{X_L{}^2}{R}$$

and since $Q = X_L/R$, we have

$$Z_o = R + QX_L$$

Since the value of R is usually very small compared to QX_L, we may write

$$\boxed{Z_o \approx QX_L} \qquad \textbf{(24.12)}$$

In the equivalent circuit of figure 24-4, the voltage across R_p and X_c must be equal:

$$e_R = e_c$$

Therefore

$$I_oR_p = i_cX_c$$

or

$$i_c = I_o\frac{R_p}{X_c}$$

However, since R_p/X_c is equal to Q, we have

$$i_c = I_oQ \qquad \textbf{(24.13)}$$

Similarly,

$$i_L = I_o\frac{R_p}{X_p} = I_oQ$$

And since Q is also equal to X_L/R, we see that

$$i_L = I_o\frac{X_L}{R}$$

must also hold for a parallel resonant circuit.

MATERIALS

- 1 Audio generator
- 1 Multimeter
- 1 Oscilloscope
- 1 Variable DC supply
- 1 Resistance decade box
- 1 1-H inductor
- 1 1-μF capacitor
- 1 5kΩ resistor
- 1 SPST switch
- 2 Sheets of linear graph paper

PROCEDURE

1. Measure and record the winding resistance of the coil (R_c).
2. Assemble the circuit shown in figure 24-5.
3. Set the audio generator for a frequency of 100 Hz.

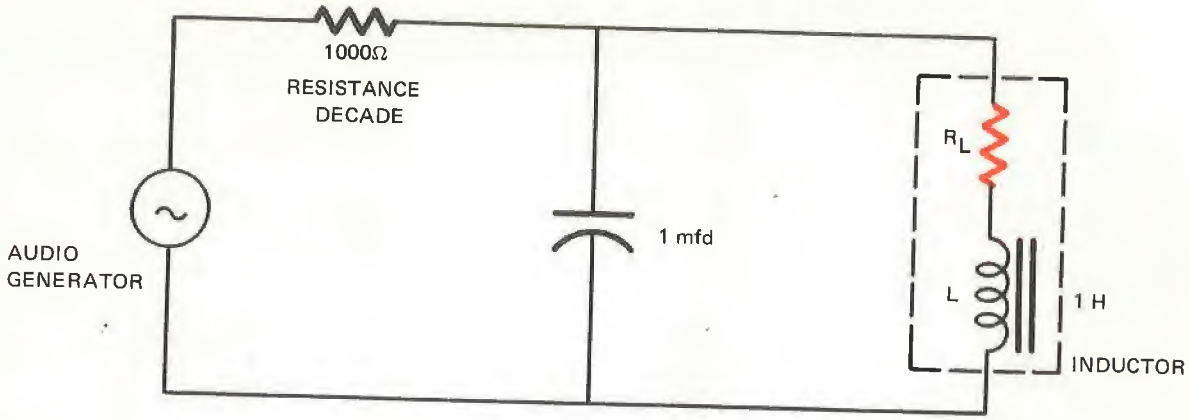


Fig. 24-5 The Experimental Circuit

4. Connect the VOM across the 1000-ohm resistor and slowly vary the generator frequency until you locate the frequency at which the VOM reading is at a minimum. Adjust the generator voltage for a VOM reading of 0.5V across the 1kΩ resistor. Record this frequency as f_m. Compute the current through the 1kΩ resistor and record it as I_o.

5. Using the VOM measure and record the voltage across the capacitor E_c. Connect the oscilloscope across the capacitor and adjust it for monitoring E_c. In the following step, adjust the generator voltage as necessary to *hold E_c constant.*

6. Connect the VOM across the 1kΩ resistor and record the voltage (E_1) for generator frequencies of 50, 60, 70, 80, 90, 100, 105, 110, 115, 120, 125, 130, 135, 140, 145, 150, 155, 160, 165, 170, 175, 180, 185, 190, 195, 200, 210, 220, 230, 240, 250, and 260 Hz. **Be very sure to hold E_c constant throughout this data run.**

7. Compute and record the total circuit current (I_T) for each frequency.

8. Compute and record the impedance of the parallel circuit for each frequency. Use $Z_T = E_c/I_T$ for this calculation.

9. On a single sheet of graph paper, plot both I_T and Z_T versus frequency.

10. Using equation 24.4, compute and record the resonant frequency f_o.

11. Using equation 24.6, compute the value of f_o' and record it.

12. From the curve plotted in step 9, determine the value of the bandwidth (BW meas).

13. Compute and record the circuit Q.

14. Using Q and f_o, compute the bandwidth (BW comp.).

15. Compute the percent difference between the two values of bandwidth.

16. Using equation 24.5, compute the value of f_o again.

17. Using equation 24.13, compute and record the value of i_c at resonance.

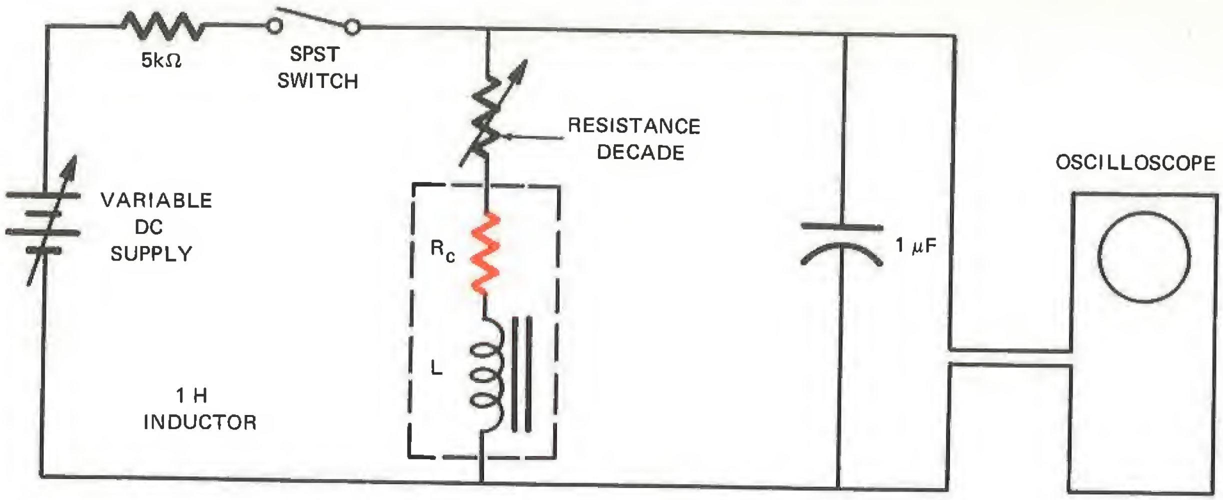


Fig. 24-6 An RLC Ringing Circuit

18. Compute X_c using $X_c = \frac{1}{2\pi f_o C}$

19. Compute i_c at resonance using $i_c = \frac{E_c}{X_c}$

20. Compute the percent difference between the two values of i_c.

21. Construct the circuit shown in figure 24-6.

22. With the SPST switch closed, set the variable DC supply for an output of about 25 volts. Set the resistance decade for a resistance of zero.

23. Slip the switch several times and observe the ringing waveform on the oscilloscope. Make an accurate sketch of the waveform.

24. Set the resistance decade to 200 ohms and repeat step 23.

25. In similar manner, repeat step 23 for decade settings of 400, 600, 800, 1000, 1200, 1400, 1600, 1800, 2000, 2200, 2400, and 2600 ohms.

26. From the data in step 25, determine the value of total circuit resistance (R_T meas) which just eliminates ringing in the circuit. (Ignore the 5kΩ current limiting resistor in this determination.)

27. Using the information given in the discussion, determine the value of *total circuit resistance* (R_T comp) necessary to achieve critical damping. (Ignore the 5kΩ current limiting resistor in this determination.)

28. Compute the percent difference between the two values of R_T.

ANALYSIS GUIDE. In considering the implications of these data, you should consider the general characteristics of parallel resonance as demonstrated by your data. In particular, you should discuss the significance and extent of agreement between your values for f_m, f_o, and f_o'.

Qty	Value
R_c	
f_m	
I_o	
E_c	
f_o	
f_o'	
BW meas	
Q	
BW comp	
% Diff BW	
f_o	
i_c	
X_c	
i_c	
% Diff i_c	
R_T (meas)	
R_T (comp)	
% Diff R_T	

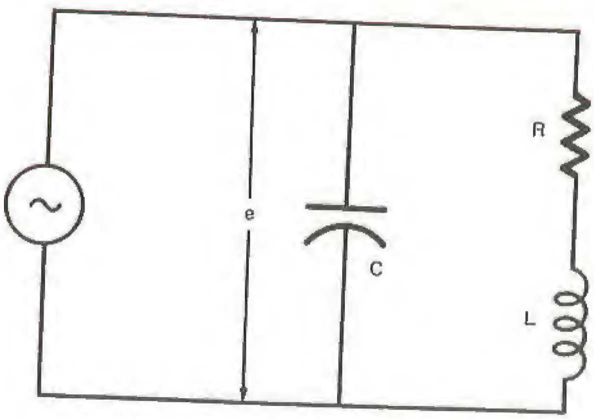
f Hz	E_1	I_T	Z_T
50			
60			
70			
80			
90			
100			
105			
110			
115			
120			
125			
130			
135			
140			
145			
150			

f	E_1	I_T	Z_T
155			
160			
165			
170			
175			
180			
185			
190			
195			
200			
210			
220			
230			
240			
250			
260			

Fig. 24-7 The Data Tables

PROBLEMS

1. Make a sketch of the total circuit current and impedance versus frequency for the circuit shown in figure 24-8.
2. What is the circuit Q in figure 24-8?



$R = 7.07k\Omega$ $L = 0.5H$

$C = 0.01\ \mu F$ $e = 100\ \text{SIN}\ \omega T$

Fig. 24-8 Circuit for Problems 1 and 2.

experiment 25 IMPEDANCE MATCHING

INTRODUCTION. In an AC circuit, as in a DC one, maximum power may be transferred from a source to a load only when the load impedance is appropriately matched to the source impedance. In this experiment we shall investigate the conditions necessary to effect maximum power transfer to a complex load.

DISCUSSION. Let us first review the simplest case of impedance matching, that being the situation in which both the source and load impedances are purely resistive. Such a case is shown in figure 25-1.

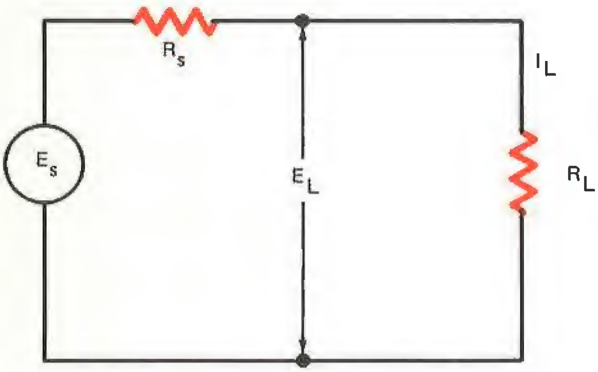


Fig. 25-1 Impedance Matching with Resistances

The power delivered to the load will be

$$P_L = I_L^2 R_L \qquad (25.1)$$

where I_L is the RMS load current and P_L is the average load power. The value of I_L is determined by the source voltage (E_s), the source resistance (R_s), and the load resistance (R_L) according to Ohm's Law,

$$I_L = \frac{E_s}{R_s + R_L}$$

Therefore, the load power can be written as

$$P_L = \frac{E_s^2 R_L}{(R_s + R_L)^2} \qquad (25.2)$$

If E_s and R_s are held constant and the value of P_L is plotted as a function of the ratio of R_L to R_s, the result will be similar to that shown in figure 25-2.

We observe from this plot that the load power reaches a maximum value of

$$P_{L\,max} = \frac{E_s^2}{4R_L}$$

when

$$\frac{R_L}{R_s} = 1$$

or

$$\boxed{R_L = R_s} \qquad (25.3)$$

This is the condition usually cited as the point of maximum power transfer for the resistive source-load circumstance. Under this condition, we say that the load is *matched* to the source.

Let us now consider the case in which the source impedance is resistive (R_s) and the load is a complex impedance (Z_L). A circuit of this type is shown in figure 25-3.

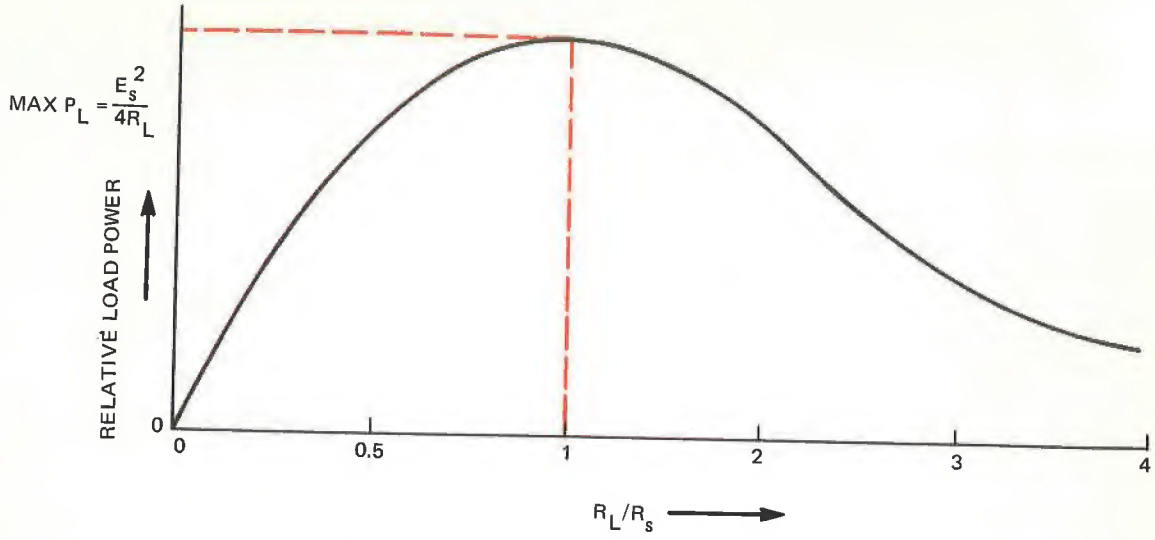


Fig. 25-2 Load Power versus the Ratio R_L/R_s

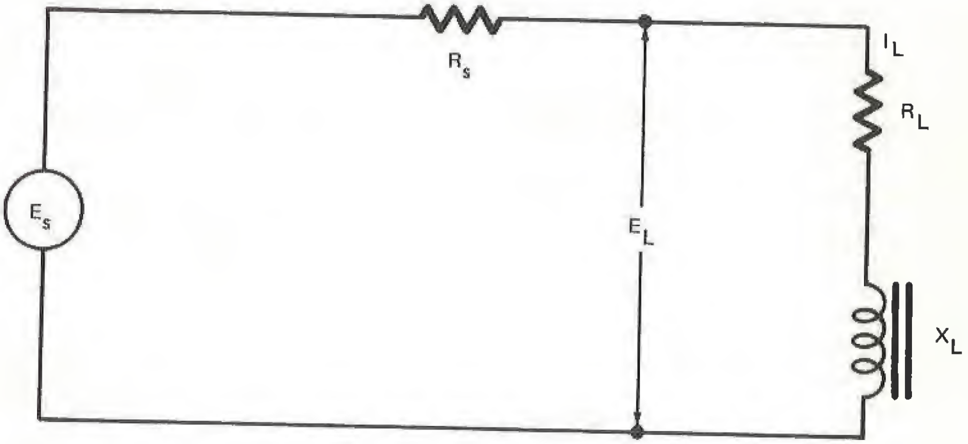


Fig. 25-3 A Resistive Source and Complex Load

In this case the load power is still

$$P_L = I_L^2 R_L$$

However, the *load current* is now given by

$$I_L = \frac{E_s}{(R_s + Z_L)}$$

Therefore,

$$P_L = \frac{E_s^2 R_L}{(R_s + Z_L)^2} \qquad (25.4)$$

Comparing this equation to equation 25.2, we see that, for a given value of R_L, the load power in the complex case will be less than in

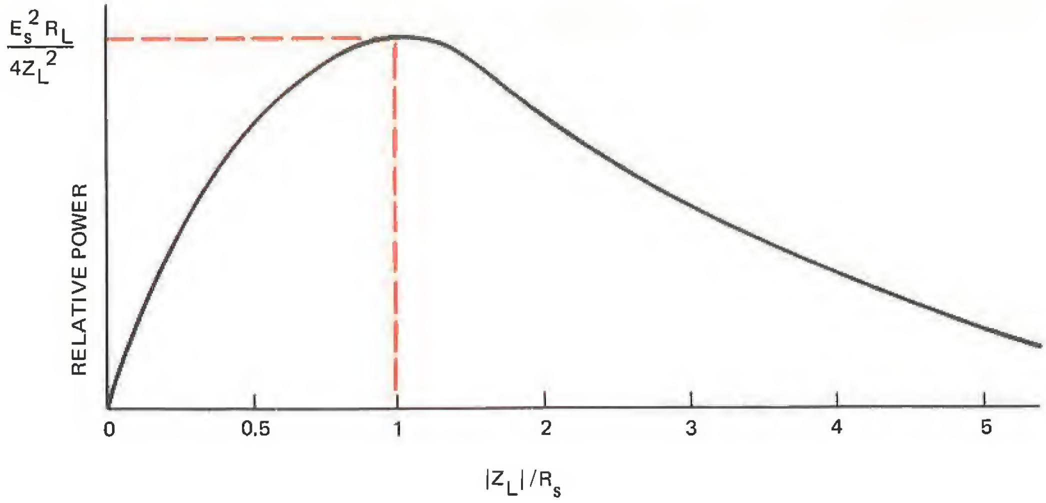


Fig. 25-4 Load power versus $|Z_L|/R_s$.

the resistive case because R_L is always less than Z.

If we plot load power versus $|Z_L|/R_S$, the result is as shown in figure 25-4. We should observe that not only is the maximum load power reduced from the resistive case, but also that the maximum occurs when

$$\boxed{|Z_L| = R_s} \qquad \textbf{(25.5)}$$

rather than when $R_L = R_s$ as before.

Let us now turn to a most general case in which both the source impedance and the load impedance are complex. Figure 25-5 shows a circuit of this type.

We should note at this point that the symbol X_L indicates the *reactance of the load* and may be either inductive or capacitive.

The value of the load power is still given by

$$P_L = I_L^2 R_L$$

But now the load current has become

$$I_L = \frac{E_s}{R_s + R_L + X_s + X_L}$$

We may, therefore, write the equation for load power in the form

$$P_L = \frac{E_s^2 R_L}{(R_s + R_L + X_s + X_L)^2}$$

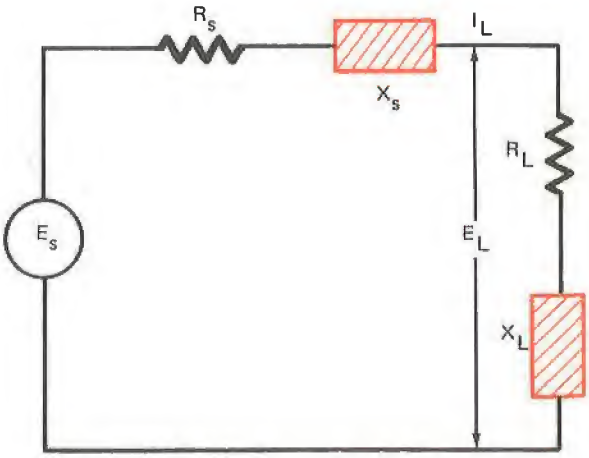


Fig. 25-5 A Complex Source and Load

Inspection of this relationship reveals that if

$$X_L = -X_s \qquad (25.6)$$

the reactive terms vanish and we have the original resistive situation. It follows, then, that maximum power will be delivered to the load when

$$\boxed{R_s \pm j\,X_s = R_L \mp X_L} \qquad (25.7)$$

This condition is termed a *conjugate match* between the source and load. It is this conjugate match situation which provides maximum power delivered to a complex load.

In some cases it is not possible to effect a conjugate match in a practical situation. At the same time we may wish to know which of several complex loads will produce the most load power. To this end we may rewrite the load current in figure 25-5 as

$$I_L = \frac{E_s}{Z_s + Z_L}$$

The load power then becomes

$$P_L = \frac{E_s{}^2 R_L}{(Z_s + Z_L)}$$

A plot of this equation reveals that maximum power is delivered when

$$\boxed{|Z_s| = |Z_L|} \qquad (25.8)$$

However, the value of the load power maximum will be reduced from that of a conjugate match if Z_s and Z_L are not complex conjugates.

In summary, we may say that maximum possible power is delivered to a complex load when the load impedance is the complex conjugate of the source impedance. If a conjugate match cannot be achieved, then an absolute value match ($|Z_s| = |Z_L|$) will produce maximum power transfer but at a lower level than a conjugate match.

MATERIALS

- 1 Oscilloscope
- 1 Multimeter
- 1 Audio generator
- 1 1-H inductor
- 2 1-μF capacitors
- 2 10-μF capacitors
- 1 Resistance decade box
- 1 100-ohm resistor
- 3 sheets of linear graph paper

PROCEDURE

1. Measure and record the value of the inductor's winding resistance (R_C).
2. Assemble the circuit shown in figure 25-6.
3. Set the audio generator frequency to 65 Hz and its output voltage control for about half the maximum value (about 1 volt across the 100Ω resistor with output terminals shorted). Adjust the oscilloscope for monitoring the generator output. Hold the generator output constant during the remainder of the experiment. It may be necessary to readjust the output control periodically to maintain constant output.

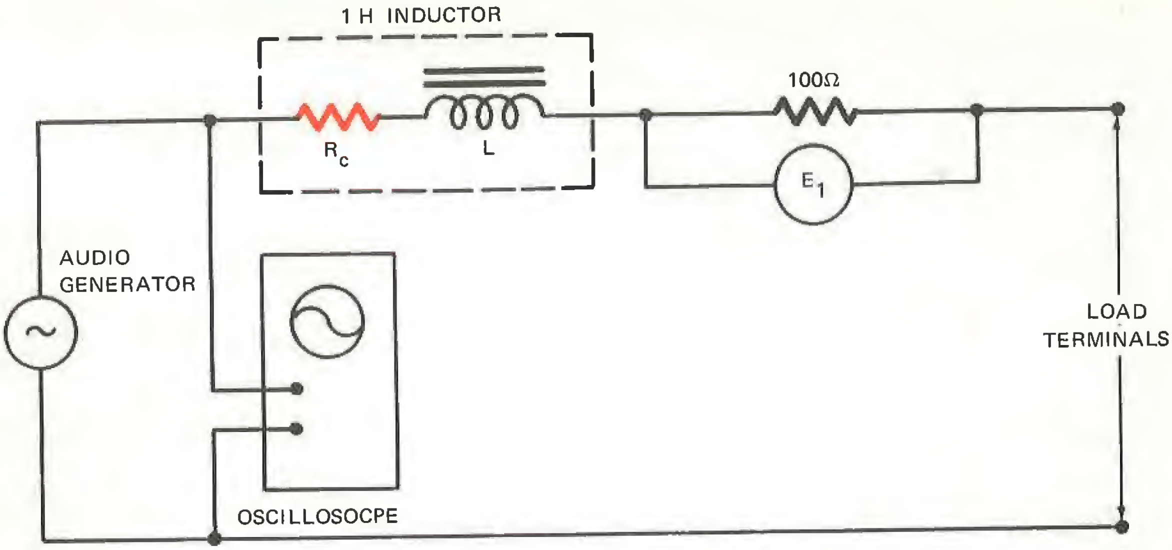


Fig. 25-6 The Experimental Source Circuit

4. Connect the first load consisting of the resistance decade only across the load terminals of the experimental source.

5. **Be sure the generator voltage remains constant!** Measure and record the voltage (E_1) across the 100-ohm resistor for each of the following resistance decade values:

(a)	0 ohms	(e)	200 ohms	(i)	400 ohms	(m)	600 ohms
(b)	50 ohms	(f)	250 ohms	(j)	450 ohms	(n)	650 ohms
(c)	100 ohms	(g)	300 ohms	(k)	500 ohms		
(d)	150 ohms	(h)	350 ohms	(l)	550 ohms		

6. Using the VOM reading and the value of the resistor, compute and record the value of the load current (I_L) for each decade setting.

7. Compute and record the value of the load power (P_L) for each decade setting.

8. Compute and record the value of the load impedance (Z_L) for each decade setting.

9. Using a 10 μF capacitor and two 1 μF capacitors, construct a circuit having a total of 12 μF capacitance. Insert this 12 mfd assembly in series with the resistance decade.

10. Repeat steps 5 through 8.

11. In like manner, repeat steps 5 through 8 with capacitances of 11 μF, 10 μF, 7 μF, 6 μF, 5 μF, and 2 μF.

12. Compute and record the following values:
    (a) The total resistance included in the source impedance (R_S)
    (b) The inductive reactance of the source impedance (X_S)
    (c) The total source impedance (Z_S)

13. On a *single* sheet of linear graph paper, plot curves of load power (P_L) versus the ratio R_L/R_s for each value of X_L.
14. On a second sheet of graph paper, plot P_L versus the ratio $|Z_L|/|Z_s|$ for each value of X_L.
15. On a third sheet of graph paper, plot P_L versus the ratio X_L/X_s using only the values in which R_s nears the value of R_L.

Resistance Decade Only

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(a)

Decade and 12 mfd cap

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(b)

Source Impedance Data

R_c	R_s	X_s	Z_s

Fig. 25-7 The Data Tables

Decade and 11 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(c)

Decade and 10 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(d)

Decade and 7 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(e)

Fig. 25-7 The Data Tables (continued)

Decade and 6 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(f)

Decade and 5 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(g)

Decade and 2 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(h)

Fig. 25-7 The Data Tables (continued)

ANALYSIS GUIDE. In the analysis of these data use your values to verify that maximum power is transferred when an absolute value match is achieved between the source and load impedance. In particular, show that a conjugate match results in a higher value of P_L than does other absolute value matches.

PROBLEMS

1. A certain generator produces a voltage of 60 sin 6280t and has an internal impedance of 600 -j 500 ohms. If we desire to transfer maximum power to a 450-ohm resistive load, how should it be done?

2. If a generator has an internal impedance of 600 +j 0 ohms, how may maximum power be coupled to a 600 +j 800-ohm load?

3. If a source has 1000 +j 0 ohms of internal impedance and is connected to a 1kΩ resistive load, maximum power transfer will occur. Is this a case of conjugate matching or simply absolute value matching? Explain why you believe your answer is correct.

experiment 26 TRANSFORMER COUPLING

INTRODUCTION. Perhaps the most commonly encountered impedance matching device in electrical applications is the *transformer.* In this experiment we shall examine the characteristics of transformer coupling.

DISCUSSION. When a current (I_1) passes through the *primary* (lefthand) *winding* of a transformer, such as the one shown in figure 26-1, a magnetic field is established around the winding.

The voltage across each turn of the primary winding will be

$$E_{n1} = \frac{E_p}{N_p} \text{ volts per turn} \qquad (26.1)$$

where E_p is voltage across the entire primary and N_p is the total number of turns in the primary winding.

Now if every line of the magnetic field which cuts the primary winding also cuts the *secondary* (righthand) *winding,* then the voltage induced in each secondary turn must equal the voltage across each primary turn. In other words, E_{n2} must equal E_{n1}. At the same time, the total secondary voltage (E_s) must be equal to the secondary voltage per turn (E_{n2}) times the total number of secondary turns (N_s).

That is

$$E_s = (E_{n2})(N_s)$$

or

$$E_{n2} = \frac{E_s}{N_s} \qquad (26.2)$$

Since E_{n2} and E_{n1} are equal, as discussed above, we may equate equations 26.1 and 26.2:

$$\frac{E_p}{N_p} = \frac{E_s}{N_s}$$

which may be rewritten in the form

$$\boxed{\frac{E_p}{E_s} = \frac{N_p}{N_s}} \qquad (26.3)$$

which tells us that the ratio of the primary to secondary voltages must equal the ratio of the primary to secondary turns.

This condition can be observed in a practical case, but will usually be only approximate because it is very difficult to design a transformer in which every magnetic line cuts

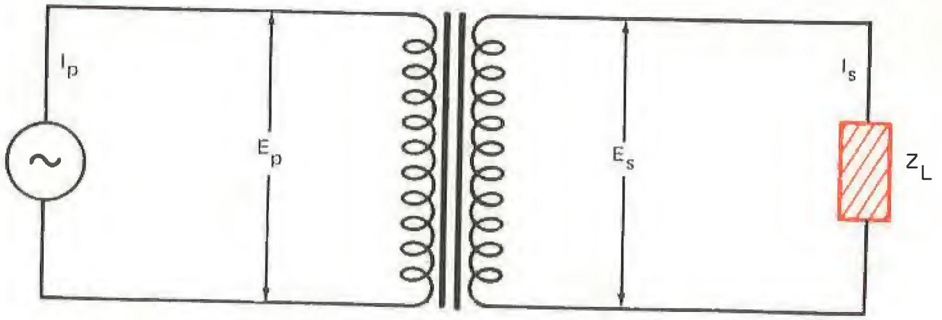


Fig. 26-1 A Transformer-Coupled Circuit

both the primary and secondary windings in an identical manner. It is, nevertheless, a very useful approximation.

If the load attached to the secondary winding is a resistance, then the power delivered from the secondary winding to the load will be

$$P_L = E_s I_s$$

Before the load was connected, the primary current (I_1) was only that amount necessary to establish the magnetic field. If we ignore the resistance of the transformer windings and any loss within the transformer core, then the original primary current (I_1) was purely inductive and, in most cases, relatively small in magnitude. When the load is connected, an additional amount of primary current (I_p') must flow because all of the load power must come from the source. The magnitude of this additional primary current must be such that

$$P_L = E_p I_p'$$

Since this product and the previous one ($E_s I_s$) are both equal to the load power, we may write

$$E_s I_s = E_p I_p'$$

or

$$\frac{I_s}{I_p'} = \frac{E_p}{E_s}$$

And since E_p/E_s is equal to N_p/N_s, we have

$$\boxed{\frac{I_s}{I_p'} = \frac{N_p}{N_s}} \qquad \textbf{(26.4)}$$

That is, the ratio of the secondary current to the *primary load current* is equal to the turns ratio.

It should be emphasized that I_p' *is not the total primary current* but only that part which flows as a direct result of connecting the load. The total primary current is the vector sum of the unloaded primary current (I_1) and the primary load current. Figure 26-2 shows the vector relationship between these two components for a resistive load.

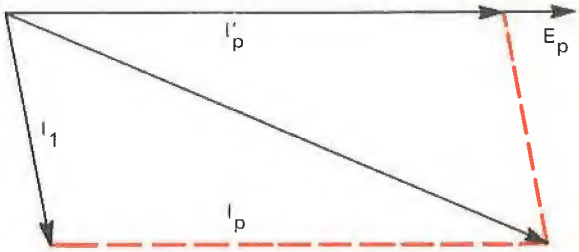


Fig. 26-2 Current in the Transformer Primary

It should be noted that if I_p' is large compared to I_1, reasonable accuracy may still be realized if we assume I_p' to be equal to the total primary current (I_p).

Returning to the original circuit, we observe that we could have expressed the relationship between the primary current (I_p) and the secondary voltage in terms of the *mutual inductance* (M) as

$$E_s = \omega M I_p$$

where ω is the angular velocity of the input voltage. That is

$$\omega = 2\pi f$$

f, being the frequency of operation.

If we now connect the load (Z_L) to the secondary winding, the total secondary current will be

$$I_s = \frac{E_s}{Z_s + Z_L}$$

where Z_s is the impedance of the secondary winding itself. Substituting the value just obtained for E_s renders

$$I_s = \frac{\omega M I_p}{Z_s + Z_L} \qquad (26.5)$$

When this secondary circuit current flows, it induces a voltage E_p' in the primary winding such that

$$E_p' = \omega M I_s$$

At the same time there is a component of the primary voltage (E_1) caused by the primary current (I_p) flowing through the primary winding impedance (Z_p)

$$E_1 = I_p Z_p$$

The net primary voltage (E_p) is, therefore, the sum of the two components E_1 and E_p'. That is

$$E_p = E_1 + E_p'$$

Since we know the values of E_1 and E_p', we may write

$$E_p = I_p Z_p + \omega M I_s$$

And if we make the substitution from equation 26.5 for I_s, we have

$$E_p = I_p Z_p + \frac{(\omega M)^2 I_p}{Z_s + Z_L}$$

Then dividing through by I_p, the equation becomes

$$\frac{E_p}{I_p} = Z_p + \frac{(\omega M)^2}{Z_s + Z_L}$$

If we recognize the ratio of E_p/I_p as being the total input impedance (Z_i) of the loaded transformer, we have

$$\boxed{Z_i = Z_p + \frac{(\omega M)^2}{Z_s + Z_L}} \qquad (26.6)$$

The term $\frac{(\omega M)^2}{Z_2 + Z_L}$ is often referred to as the *reflected impedance* of the load.

When $Z_s + Z_L$ is predominantly resistive (R_L'), then the reflected impedance is also resistive. And, since the primary winding impedance is almost purely inductive, the reflected impedance constitutes virtually all of the resistive portion of the input impedance. Under these conditions, we may write

$$R_i = \frac{(\omega M)^2}{R_L'}$$

Multiplying both sides by $R_i/(\omega M)^2$ gives us

$$\frac{R_i^2}{(\omega M)^2} = \frac{R_i}{R_L'}$$

Then multiplying both numerator and denominator on the left by I_p^2, we have

$$\left(\frac{I_p R_i}{\omega M I_p}\right)^2 = \frac{R_i}{R_L'}$$

Since $\omega M I_p$ is equal to E_s, and $I_p R_i$ is equal to E_p when the reactive portion of I_p is small (see figure 26-2), then we may write

$$\boxed{\left(\frac{E_p}{E_s}\right)^2 = \frac{R_i}{R_L'}}$$

Finally, since E_p/E_s is equal to N_p/N_s, the turns ratio, we have

$$R_i = \left(\frac{N_p}{N_s}\right)^2 R_L' \qquad (26.7)$$

That is to say, the input resistance of a transformer is equal to the square of the turns ratio times the load resistance when the winding impedances are negligible.

If we use the symbol Z_R for the reflected impedance, then we have

$$Z_R = \frac{(\omega M)^2}{Z_s + Z_L} \qquad (26.8)$$

There are several special cases which are of interest. For example, we observe that if the secondary is open circuited ($Z_s + Z_L \rightarrow \infty$) then Z_R approaches zero. Similarly:

a) If $Z_s + Z_L$ is resistive, then Z_R is also resistive.

b) If $Z_s + Z_L$ is inductive, then Z_R is capacitive.

c) If $Z_s + Z_L$ is capacitive, then Z_R is inductive.

Equations 26.3, 26.4, 26.7, and 26.8 illustrate the four main transformer applications. They are:

1. To step alternating voltages up or down in magnitude. (Eq. 26.3)
2. To step alternating currents up or down in magnitude. (Eq. 26.4)
3. To step load resistances up or down for impedance matching purposes. (Eq. 26.7)
4. To convert impedance characteristics from inductive to capacitive or visa versa for matching or tuning purposes. (Eq. 26.8)

There are also a variety of other applications such as isolating one circuit from another and waveshaping which will not be dealt with specifically in this experiment. We will, however, experimentally verify the four main principles outlined above.

MATERIALS

1 Audio generator
1 Multimeter
1 1-μF capacitor
1 1-H inductor
1 100-ohm resistor
1 AC ammeter
1 10-μF capacitor
1 1:1 transformer
1 Variable transformer
1 Sheet of linear graph paper
1 Oscilloscope

PROCEDURE

1. Measure and record the primary and secondary winding resistances, R_p and R_s.
2. Assemble the circuit shown in figure 26-3.
3. Using the VOM, set the variable transformer for a primary voltage, E_1, of about 30 volts RMS. Record this value of E_1 in the Data Table.

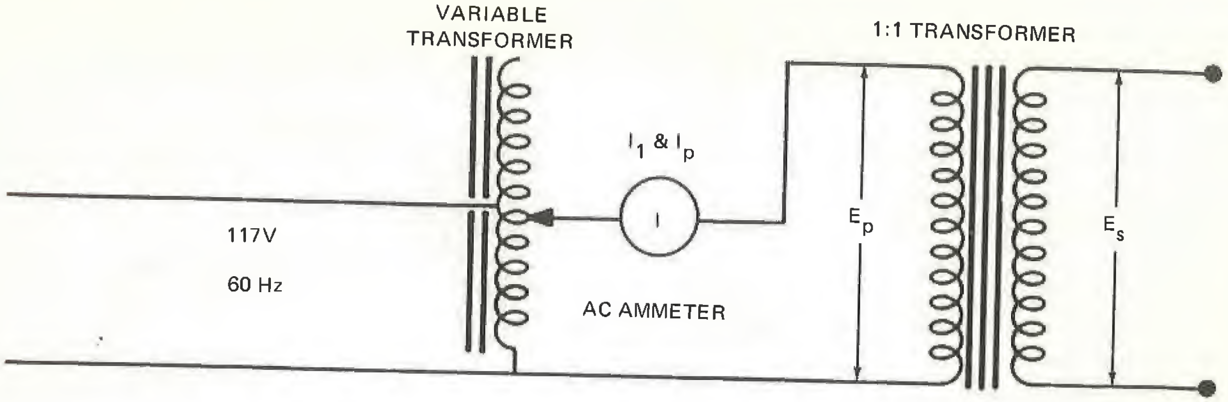


Fig. 26-3 The First Experimental Circuit

4. Measure and record the values of E_s and I_1 in the Data Table.

5. Compute the transformer turns ratio using E_1 and E_s.

6. Compute the primary winding impedance (Z_p) using E_p and I_1.

7. Using the primary winding impedance (Z_p) and resistance (R_p), compute the primary winding reactance (X_p).

8. Using R_p and X_p, compute the phase angle (θ_p) between the primary voltage and the magnetizing current (I_1).

9. Connect the 100-ohm resistor to the secondary winding and recheck the primary voltage to insure that E_p is still set at the value chosen in step 3. Readjust the variable transformer if necessary and record E_p again in the "100-ohm load" Data Table.

10. Record the new values of I_p and E_s.

11. Using E_s and R_L, compute and record I_s.

12. On a sheet of graph paper, plot the value of I_1 measured in step 4 at the angle computed in step 8. Then plot I_p in such a way as to allow you to determine the primary load current I_p' (see figure 26-2). Record the value of I_p' thus determined.

13. Compute and record the value of the turns ratio using I_s and I_p'.

14. Compute the percent difference between the two values of turns ratio.

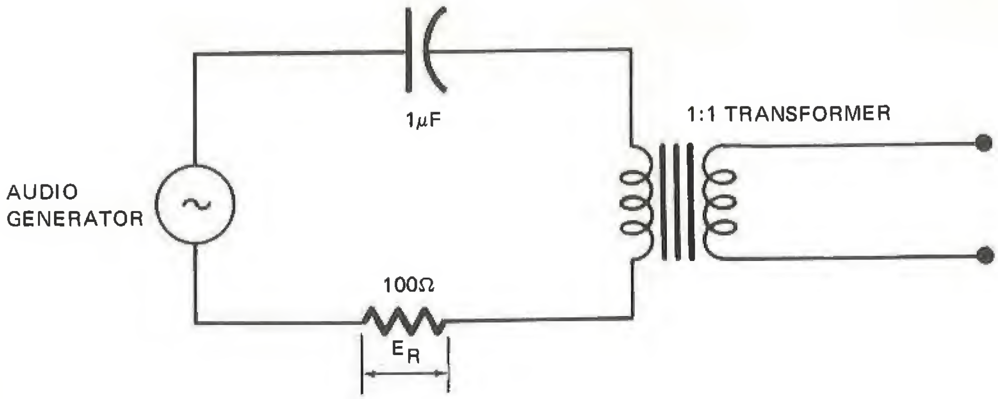


Fig. 26-4 The Second Experimental Circuit

15. Compute the input impedance (Z_i) using E_p and I_p.

16. Compute the value of the reflected load impedance using $Z_R = E_p/I_p'$.

17. Compute the percent difference between R_i and Z_R.

18. Assemble the circuit shown in figure 26-4.

No Load

Qty	Value
R_p	
R_s	
E_1	
E_s	
I_1	
N_p/N_s	
Z_p	
X_p	
θ_p	

100-ohm Load

Qty	Value
E_p	
I_p	
E_s	
I_s	
I_p'	
N_p/N_s	
% Diff N_p/N_s	
Z_i	
Z_R	
% Diff Z_i, Z_R	

L & C Load

Qty	Value
f_1	
f_2	
f_3	

Fig. 26-5 The Data Tables

19. **Compute but do not record** the resonant frequency of the primary circuit using the values of C and X_p from step 7.

20. Set the generator for maximum output at the frequency computed above.

21. Connect the oscilloscope to indicate the voltage (E_R) across the 100-ohm resistor.

22. Adjust the frequency as necessary to get a maximum reading of E_R. Record the frequency (f_1) at which E_R is at maximum.

23. Connect a 10-μF capacitor across the secondary and readjust the generator frequency for maximum E_R. Record the new resonant frequency (f_2).

24. Replace the 10-μF capacitor with a 1-H inductor and repeat step 23. Record the result as f_3.

ANALYSIS GUIDE. In analyzing these data you should be primarily concerned with using your data to verify the utility of a transformer in the four main applications given in the discussion.

In particular consider the effects of the inductive and capacitive loads. What was the nature (inductive or capacitive) of the reflected impedance? How does your data indicate this? Did the reflected impedance act effectively in series or in parallel with the primary winding impedance?

PROBLEMS

1. What turns ratio is necessary to step 117 volts, 60 Hz up to 700 volts?

2. What size fuse should be used in the primary of a 110-volt transformer if the secondary load is to be 3A at 6.3 volts? (Assume a perfect transformer.)

3. What turns ratio is required to match a source having an internal resistance of 10kΩ to a 4-ohm load?

4. What would be the *open circuit* source voltage in problem 3 if load voltage is 3.2 volts?

experiment 27 TRANSFORMER MATCHING

INTRODUCTION. As far as electronics is concerned, impedance matching consists of connecting an electronic source with a specified source impedance to a specified load. In this experiment we shall consider only one matching device – the *nonresonant transformer.*

DISCUSSION. If we connect an ideal transformer, such as the one depicted in figure 21-1, to a practical voltage source, the voltage across each primary turn of the transformer will be

$$e_p = \frac{V_p}{N_p} \qquad (27.1)$$

where V_p is the voltage across the entire primary winding, and N_p is the number of primary turns.

If the transformer has unity coupling (every magnetic line which cuts a primary turn also cuts a secondary turn), then the voltage per turn in the secondary must equal the voltage per turn in the primary. That is

$$e_s = e_p \qquad (27.2)$$

Moreover, the total secondary voltage (V_L) will be equal to the secondary volts per turn times the number of secondary turns (N_s), or

$$V_L = e_s N_s$$

which may be rewritten as

$$e_s = \frac{V_L}{N_s} \qquad (27.3)$$

And since by equation 27.2

$$e_p = e_s$$

we have

$$\frac{V_p}{N_p} = \frac{V_L}{N_s}$$

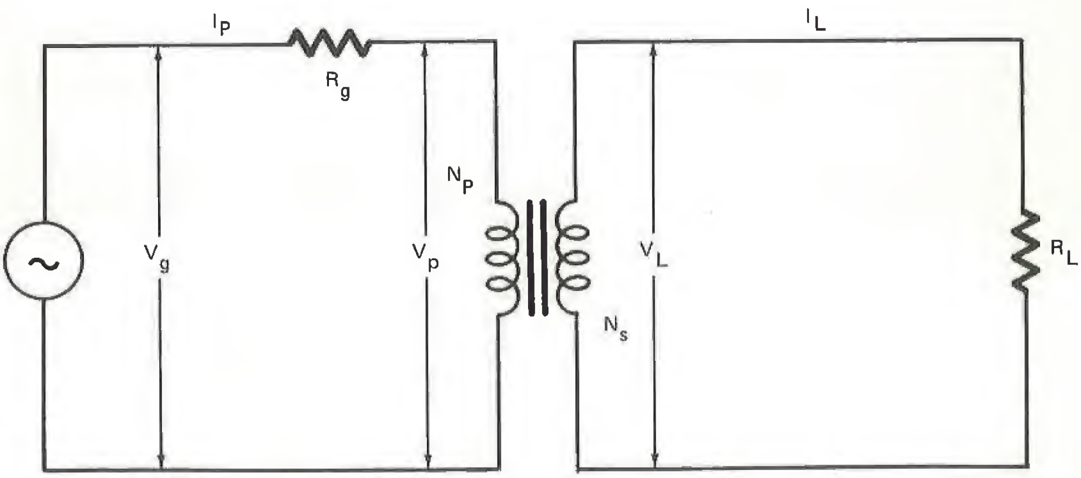


Fig. 27-1 A Transformer-coupled Load and Source

or

$$\frac{V_p}{V_L} = \frac{N_p}{N_s} \qquad (27.4)$$

We may, therefore, evaluate the turns ratio of a transformer by measuring the primary and secondary voltages.

If the transformer used approaches the ideal (has negligible losses), then the energy applied to the primary winding must approach the energy consumed in the secondary circuit. That is

$$P_p t = P_s t$$

where P_p and P_s are the primary and secondary power respectively and t is time. Because the time factor affects both the primary and secondary equally, we may write

$$P_p = P_s$$

And since (in the ideal case) $P = VI$, we have

$$V_p I_p = V_L I_L$$

for the circuit shown in figure 27-1. We can rewrite this relationship as

$$\frac{I_L}{I_p} = \frac{V_p}{V_L} \qquad (27.5)$$

And referring to equation 27.4,

$$\frac{I_L}{I_p} = \frac{N_p}{N_s} \qquad (27.6)$$

Let us now focus our attention on the load resistance, R_L. It should be apparent that

$$V_L = I_L R_L$$

Substituting this relationship into equation 27.5 renders

$$\frac{I_L}{I_p} = \frac{V_p}{I_L R_L}$$

or

$$\frac{I_L{}^2}{I_p} = V_p \left(\frac{1}{R_L}\right)$$

Then, dividing both sides by I_p gives us

$$\frac{I_L{}^2}{I_p{}^2} = \frac{V_p}{I_p}\left(\frac{1}{R_L}\right)$$

At this point, we recognize the ratio V_p/I_p as the input impedance (R_p) of the transformer. Therefore, we have

$$\left(\frac{I_L}{I_p}\right)^2 = \frac{R_p}{R_L}$$

From equation 27.6, we may see that

$$\left(\frac{N_p}{N_s}\right)^2 = \frac{R_p}{R_L}$$

which we can rewrite in the form

$$\boxed{R_p = \left(\frac{N_p}{N_s}\right)^2 R_L} \qquad (27.7)$$

This equation is the whole basis for using a transformer as an impedance matching device.

Notice that by choosing the proper value for N_p/N_s (that is, choosing the proper transformer), we may match any load value to any source provided that the load is resistive and the transformer approaches the ideal.

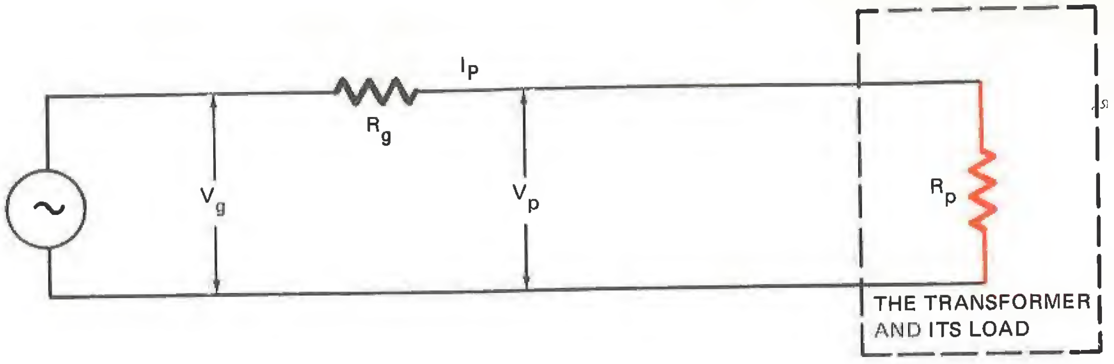


Fig. 27-2 An Equivalent for the Original Circuit

Based on equation 27.7, we can redraw the original circuit as shown in figure 27-2.

The primary current will be

$$I_p = \frac{V_g}{R_g + R_p}$$

and the power delivered to R_p will be

$$P_L = I_p^2 R_p = \frac{V_g^2 R_p}{(R_g + R_p)^2}$$

Since R_p and R_L are related, as indicated by equation 27.7, we may rewrite the power as

$$P_L = \frac{V_g^2 \left(\frac{N_p}{N_s}\right)^2 R_L}{\left[R_g + \left(\frac{N_p}{N_s}\right)^2 R_L\right]^2}$$

This relationship may be simplified somewhat by dividing both the numerator and the denominator by $(N_p/N_s)^2$.

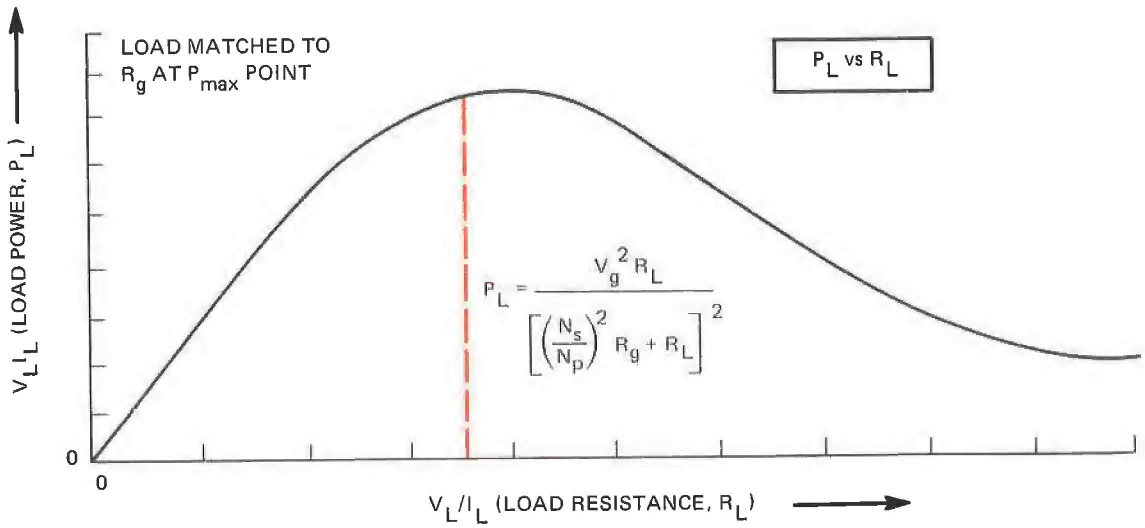


Fig. 27-3 Load Power versus Load Resistance

$$P_L = \frac{V_g^2 R_L}{\left[\left(\frac{N_s}{N_p}\right)^2 R_g + R_L\right]^2} \qquad (27.8)$$

Returning to the original circuit (fig. 27-1), we observe that

$$P_L = V_L I_L \text{ and } R_L = V_L/I_L$$

which makes both P_L and R_L readily measurable at the load.

If we plot P_L (or $V_L I_L$) versus R_L (or V_L/I_L) using equation 27.7, the result will be a curve similar to figure 27-3.

At the point where the power is at maximum, the value of R_p equals that of R_g, and, of course, (by equation 27.7)

$$\left(\frac{N_p}{N_s}\right)^2 R_L = R_g \qquad (27.9)$$

Under this condition (maximum load power), we say that the load (R_L) is *matched* to the source (R_g) by the transformer turns ratio (N_p/N_s).

Notice that equation 27.9 clearly indicates that any value of the load resistance can be matched to any source resistance if the appropriate transformer turns ratio is used.

MATERIALS

1 Audio generator
1 Multimeter
1 Resistance decade box
1 Audio transformer (Stancor type TA-28 or equivalent)
1 Sheet of linear graph paper

PROCEDURE

1. Assemble the circuit shown in figure 27-4.
2. Set the audio generator for maximum output at 1 Khz.
3. Measure and record:
   a) The primary voltage (V_p)
   b) The total secondary voltage (V_{s1})
   c) The center tap secondary voltage (V_{ct})

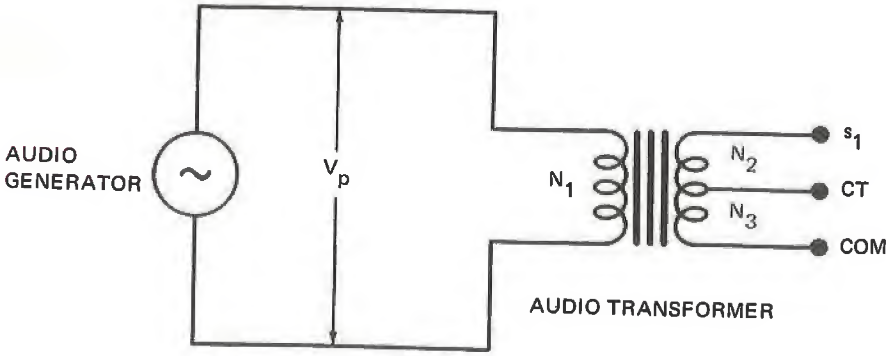


Fig. 27-4 Determining the Turns Ratios

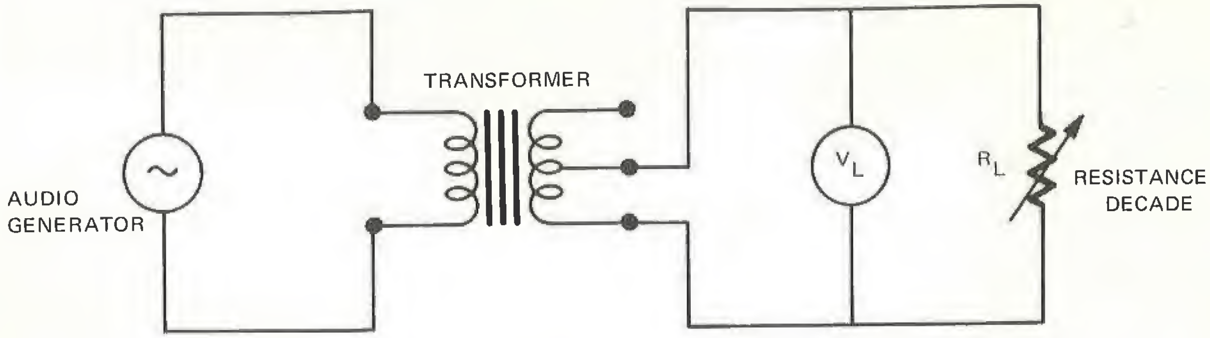


Fig. 27-5 The Experimental Circuit

4. Compute and record the four different values of turns ratio indicated in the Data Table.
5. Using the largest value of turns ratio between the source and load, connect the circuit shown in figure 27-5.
6. Measure and record the RMS load voltage for each value of load resistance shown in the Data Table (Fig. 27-6).
7. Rearrange the circuit so that the source and load are connected through the second largest turns ratio and repeat step 6.
8. Similarly, repeat step 6 for the two remaining turns ratios.
9. For each measurement, compute the load power using V_L and R_L. Record these values in the Data Table column headed "$V_L I_L$."
10. On a single sheet of linear graph paper plot the load power ($V_L I_L$) versus the load resistance (V_L/I_L) for the four values of turns ratio.

Turns Ratio Data

V_p	V_{s1}	V_{ct}	N_1/N_2	$N_1/(N_2+N_3)$	N_2/N_3	N_3/N_1

ANALYSIS GUIDE. In analyzing the data from this experiment, you should consider the following points:

a) Was maximum power transfer achieved in each of the four cases?
b) Was the value of the maximum power the same in each case?
c) Was the value of R_L the same in each case when maximum power was reached?

PROBLEMS

1. An electronic amplifier has an output impedance of 2k ohms. What transformer turns ratio is required to match this amplifier to an 8-ohm load?
2. The amplifier in problem 1 has a signal output voltage of 9 volts RMS. What is the load current? What is the load power?
3. Two identical amplifier stages are coupled together using a transformer. Each stage is known to have an output resistance of 5k ohms and an input resistance of 1k ohms. What is the required turns ratio for maximum power transfer?

Load Power Data

Load Resistance V_L/I_L (ohms)	Turns ratio =		Turns ratio =		Turns ratio =		Turns ratio =	
	V_L	V_LI_L	V_L	V_LI_L	V_L	V_LI_L	V_L	V_LI_L
0								
50								
100								
150								
200								
300								
400								
500								
600								
700								
800								
900								
1000								
1200								
1400								
1600								
1800								
2000								
2400								
2800								
3200								
3600								
4000								
4500								
5000								

Fig. 27-6 The Data Table

ELECTRONICS

ELECTRICITY

APPENDIX

DELMAR PUBLISHERS, MOUNTAINVIEW AVENUE, ALBANY, NEW YORK 12205

A. LABORATORY REPORT WRITING

There are a number of different forms that a technical laboratory report may take. The forms proposed here are intended to meet the needs of these experiments and should not be considered to be universally applicable.

I. THE INFORMAL REPORT. In reporting the results of these experiments, it may be convenient to write an informal type of laboratory report. Informal reports are normally due at the start of the laboratory period following the period during which the experiment was performed. A report outline which you may wish to follow is given below.

A. **Cover Page.** The cover page should include:
   1. Your name.
   2. Your partner's name.
   3. Date the experiment was performed.
   4. Experiment title and number.

B. **Data Section.** The data section should include:
   1. A neat drawing of the experiment setup.
   2. A list of equipment used, including the manufacturer's name, model number, and serial number.
   3. Measured and calculated data in tabular form.
   4. Curves.

C. **Analysis Section.** The analysis section should contain a satisfactory technical discussion of the data. It should, in general, include brief discussions of each of the points mentioned in the "Analysis Guide" and the solutions to any problems given at the end of the experiment.

II. THE FORMAL REPORT. You may be required to write and submit a formal laboratory report on some of the experiments that you have performed. All formal reports should be submitted in a satisfactory report folder, and are normally due about 1 week after the time that the experiment was performed. The formal report should include the following:

A. **Title Page.** The title page should contain the following:
   1. Title of the experiment.
   2. Name of the person making the report.
   3. Date the experiment was performed.

B. **Introduction Section.** The introduction should consist of a paragraph which sets forth the technical objective of the experiment.

C. **Theory Section.** The theory section should include a brief discussion of the theory which is pertinent to the particular experiment.

D. **Method of Investigation Section.** The method of investigation should include the following:
   1. A neat drawing of the experimental setup.
   2. A brief outline of the experimental procedure.
   3. A brief outline of the calculations to be made.
   4. A brief discussion of how the calculations and measurements are to be compared.

E. **Equipment List.** The equipment list should contain every item of equipment used. It should show the manufacturer's name, the model number, and the serial number of every item.

F. **Data Section.** The data section should include a smooth copy of the following:
   1. All measured values in tabular form.
   2. All computed values in tabular form.
   3. All curves.

G. **Sample Computations Section.** This section should include a smooth sample of each type of calculation made.

H. **Analysis Section.** The analysis section should include a discussion of each of the following points:
   1. How valid is the data?
   2. What are the probable sources of error?
   3. What are the probable magnitudes of the different errors?
   4. How could the errors be reduced?

I. **Rough Data Section.** This section is provided to contain all work not presented elsewhere in the report. It should contain such items as:
   1. Notes taken from reference material.
   2. The actual calculations performed.
   3. The actual rough experimental data.

As you have no doubt already concluded, the writing of a formal laboratory report is by no means quick or easy. You should remember however that a technician is frequently judged on the quality of his reports. Therefore, it is wise to make each report as good as possible.

B. SAMPLE EXPERIMENT

Experiment 3 HALFWAVE RECTIFIERS

INTRODUCTION. Because of its simplicity and economy, the halfwave rectifier is one of the most common rectifier circuits. In this experiment we shall examine the distribution of voltage and current within such a rectifier circuit.

DISCUSSION. The halfwave rectifier is rarely used without a filter circuit. However, in this experiment we shall be primarily concerned with the action or the rectifier only; consequently, we shall consider it without a filter. Figure 3-1 shows a simple halfwave circuit.

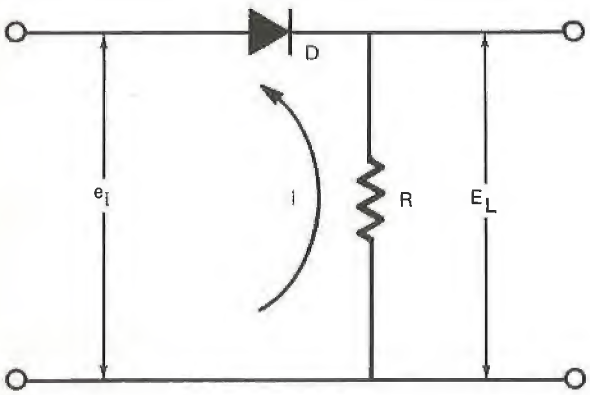


Fig. 3-1 A Simple Halfwave Rectifier Circuit.

The forward resistance of the diode will, of course, be nonlinear. However, the analysis of the circuit will be simplified if this quantity is assumed to have a fixed value (r_D). Such an assumption will introduce only a small error into our final results. Also, the diode may present a finite back resistance which would affect the final results. For the sake of simplicity, we shall assume this effect to be negligible.

If a sinusoidal voltage is applied to the input, the circuit current will have a waveshape similar to that shown in figure 3-2. If we further specify that the input voltage is to be given by the relationship,

$$e_i = E_M \text{ Sin } \omega t$$

then, considering the assumptions given above, we can write equations for the current:

$$i = I_M \text{ Sin } \omega t \quad \text{if} \quad 0 \leqslant \omega t \leqslant \pi$$

$$i = 0 \quad \text{if} \quad \pi \leqslant \omega t \leqslant 2\pi$$

Examination of the circuit reveals that the maximum current will be

$$I_M = \frac{E_M}{r_D + R} \tag{3.1}$$

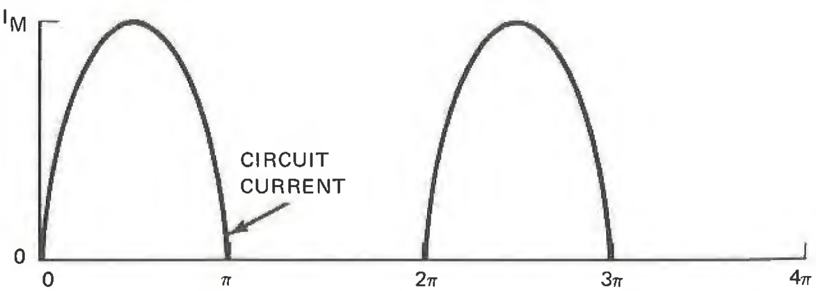


Fig. 3-2 The Circuit Current.

Consequently, we have

$$i = \frac{E_M}{r_D + R} \text{Sin } \omega t \quad \text{if} \quad 0 \leqslant \omega t \leqslant \pi$$

and

$$i = 0 \quad \text{if} \quad \pi \leqslant \omega t \leqslant 2\pi$$

The DC current which will flow in the circuit will be the average of the current taken over the two intervals. That is,

$$I_L = \frac{1}{2\pi} \int_0^{2\pi} i \, d\, \omega t$$

However, since i = 0 in the interval $\leqslant \omega t \leqslant 2\pi$, we have,

$$I_L = \frac{1}{2\pi} \int_0^{\pi} \frac{E_M}{r_D + R} \text{Sin } \omega t \, d\, \omega t \qquad (3.2)$$

Taking the integral renders

$$I_L = \frac{1}{2\pi} \left[\frac{E_M}{r_D + R} \text{ Cos } \omega t \right]_0^{\pi}$$

and evaluating the equation at the indicated limits we have,

$$I_L = \frac{E_M}{\pi (r_D + R)} \qquad (3.3)$$

Finally, we know that

$$E_L = I_L R$$

Consequently,

$$E_L = \frac{E_M R}{\pi (r_D + R)} \qquad (3.4)$$

which is, of course, the DC voltage across the load.

MATERIALS.

1 Silicon diode

1 110-volt, 60-cycle source

1 Oscilloscope

1 Vacuum tube voltmeter

1 5kΩ, 1W load resistor

PROCEDURE.

1. Construct the circuit shown in figure 3-1.
2. Measure and record the peak value of the input voltage (E_m).
3. Measure and record the peak value of the voltage across the 5kΩ resistors (E_{pk}).
4. Using the data from steps 2 and 3, compute the approximate value of r_D.
5. Measure and record the DC voltage across the 5kΩ load resistor.

6. Using the data from steps 2, 3, and 4 and equation 3.4 from the discussion, compute the value of the DC voltage which should appear across the 5kΩ resistor.
7. Compute the percent difference between the two values of DC voltage from steps 5 and 6.
8. View and record as accurately as possible the waveshape of:
   a) The input voltage
   b) The voltage across the diode
   c) The voltage across the 5kΩ resistor
9. Using the value measured in step 5, compute the value of I_L.
10. Using the data from steps 2, 3, and 4 and equation 3.3 from the discussion, compute the value of I_L.
11. Compute the percent difference between the two values of I_L arrived at in steps 9 and 10.

Quantity	E_m	E_{pk}	r_D	E_L	I_L
Measured Values					
Computed Values	X	X	X		
Difference	X	X	X		

Fig. 3-3 The Data Table

ANALYSIS GUIDE. In the analysis of these data, you should discuss the reasons for the differences between the values of voltages and currents that were encountered in the experiment. In particular, you should discuss the factors to which these differences could be attributed.

PROBLEMS.

1. A certain diode has a plate resistance (r_D) of 500 ohms. What would be the values of the DC voltage and current supplied to a 30kΩ load if the load and diode are connected in series across a 117 VRMS, 60 Hz line?
2. In the problem above, what value of DC voltage would appear across the diode?

C. SAMPLE REPORT

ELECTRONICS TECHNOLOGY

Course No. E-1224

Experiment No. 3

HALFWAVE RECTIFIERS

Informal report

Reported by:	John Doe
Co-worker:	Alex Smith

Date:______________________

PROCEDURE

1. The circuit in figure 1 was constructed.
2. Various waveforms were compared to calculated values.
3. The measurements were compared to calculated values.

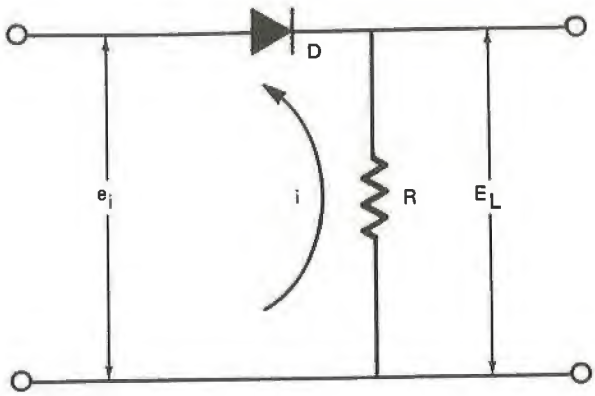


Fig. 1 The Experimental Circuit.

EQUIPMENT

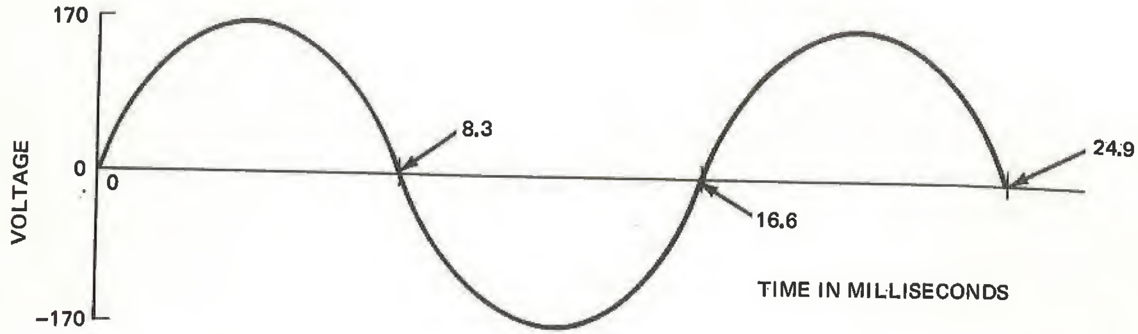
1 Silicon diode, General Electric type 1N1692
1 Resistance decade, Industrial Instruments, Model DR-50, No. 8305
1 Vacuum Tube Voltmeter, Radio Corp. of America, Model WV-77E, No. 40378
1 Oscilloscope, Tektronix Type 535, No. 9463

DATA

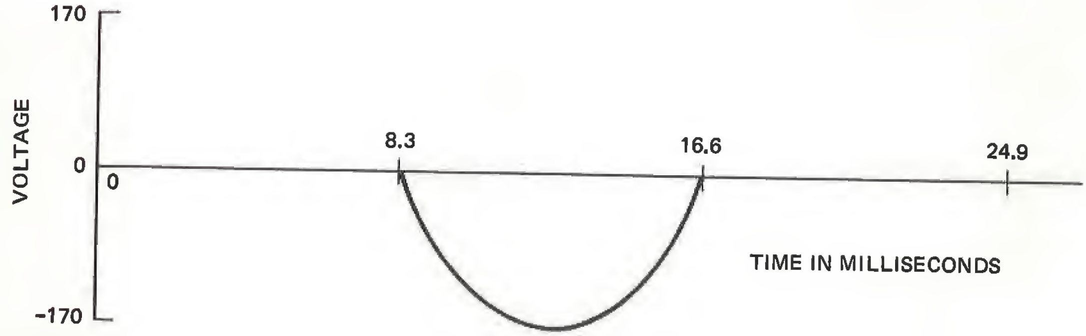
Quantity	E_m	E_{pk}	r_D	E_L	I_L
Measured Values	170 Volts	169 Volts	29.6 Ohms	52.3 Volts	10.45 mA
Computed Values				54.1 Volts	10.81 mA
Difference				3.45%	3.44%

Fig. 2 Data Table

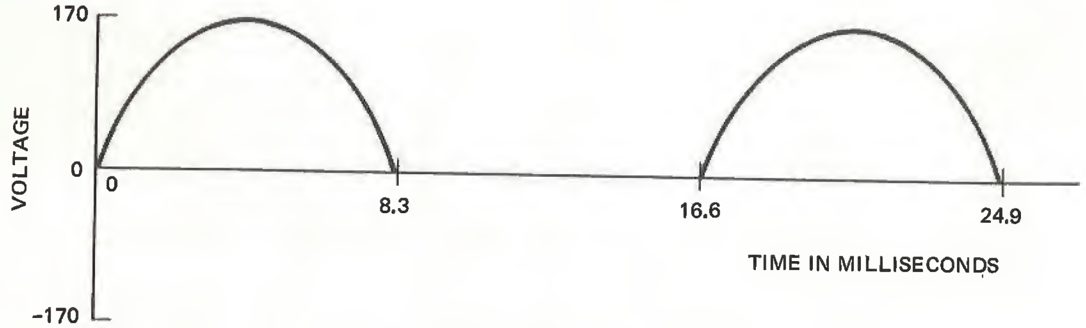
CURVES: See attached graph sheet. (See page 181.)



(A) VOLTAGE ACROSS THE INPUT



(B) VOLTAGE ACROSS THE DIODE



(C) VOLTAGE ACROSS THE 5K RESISTOR

WAVESHAPES FROM THE CIRCUIT

ANALYSIS OF RESULTS

The values measured in this experiment agreed very well with those predicted by the equations given in the discussion, the difference between the two being about 3.5%. The causes of this difference are not obvious, but could have arisen from any combination of several sources. For example, the voltage measurements used to compute the diode's forward resistance were taken with the oscillosocpe and were within 1% of each other. A very small error in measuring either of these voltages would introduce a large error into the value computed for r_D. Fortunately, however, because of the great difference in the value between r_D and the 5K load, even large errors in the value of r_D would introduce only small errors in the values of E_L and I_L. The instruments used to make the measurements should have been accurate to within about 5% of the indicated values. Since the differences between computed and measured values were less than 5%, it seems reasonable to conclude that the errors could be attributed almost entirely to the instruments.

The waveshapes that were recorded indicate that the circuit was operating satisfactorily and that the assumptions made in the discussion were valid as far as this experiment was concerned.

PROBLEM SOLUTIONS

1. Given:

   $E_i = 117$ VRMS $\qquad r_D = 500$ ohms

   $f = 60$ Hz $\qquad R = 30$K ohms

   Find:

   E_L, I_L

$$E_M = \sqrt{2}\ E_i = 1.414 \times 117 = 166 \text{ volts}$$

$$I_L = \frac{E_M}{\pi (r_D + R)} = \frac{166}{3.14\ (500 + 30000)} = \frac{166}{9.15 \times 10^4} = \underline{\underline{1.81 \text{ mA}}}$$

$$E_L = I_L R = 1.81 \times 10^{-3} \times 3 \times 10^4 = \underline{\underline{54.3 \text{ volts}}}$$

2. Since the DC value of the input is zero, the voltage across the diode (E_D) plus the voltage across the load (E_L) must also be zero.

$$E_D + E_L = 0$$

$$E_D = -E_L = \underline{\underline{-54.3 \text{ volts}}}$$

D. SAMPLE REPORT

ELECTRONICS TECHNOLOGY

Course No. E-1224

Experiment No. 3

HALFWAVE RECTIFIERS

Formal report

Reported by: John Doe
Co-worker: Alex Smith

Date: ____________________

INTRODUCTION.

In this experiment the operation of a simple halfwave rectifier was examined. The extent to which its performance could be predicted by a mathematical model was also investigated. The techniques employed in this experiment would be valid for any simple halfwave rectifier in which no filter was employed, provided that the forward resistance of the diode was small compared to the load resistance, and that the reverse resistance of the diode was large compared to the load resistance. Normally, this would include most vacuum, gaseous, and solid-state diodes.

METHOD OF INVESTIGATION

A. Procedure

1. The circuit shown in figure 1 was constructed.
2. Using an oscilloscope, the peak voltage at the input and across the 5kΩ load resistor was recorded.
3. Using circuit values and the voltages measured above, the value of the diode's forward resistance (r_D) was approximated.
4. Using a VTVM, the DC voltage across the 5kΩ load was recorded.
5. The load current was determined from the data taken above.
6. Waveshapes at various points in the circuit were viewed and sketched.
7. The values of DC load voltage and current were computed using a mathematical model.
8. The measured and computed values of load voltage and current were compared.

B. Circuit Diagram

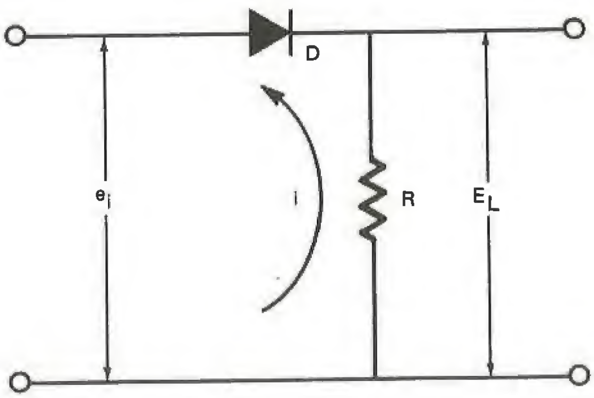


Fig. 1 The Experimental Circuit

RESULTS

A. Data

1. Nameplate data of equipment

1 Silicon diode	General Electric Type 1N1692
1 Resistance decade	Industrial Instruments Model DR-50 No. 8305
1 VTVM	Radio Corp. of America Model WV-77E No. 40378
1 Oscilloscope	Tektronix Type 535 No. 9463

2. Observed and calculated data

Quantity	E_m	E_{pk}	r_D	E_L	I_L
Measured Values	170 Volts	169 Volts	29.6 Ohms	52.3 Volts	10.45 mA
Computed Values				54.1 Volts	10.81 mA
Difference				3.45%	3.44%

Fig. 2 Data Table

B. Sample Calculations

1. Diode resistance

$$\frac{r_D}{E_M - E_{pk}} = \frac{R}{E_{pk}}$$

$$\frac{r_D}{170 - 169} = \frac{5,000}{169}$$

r_D = 29.6 ohms (by sliderule)

2. DC voltage across the load

$$E_L = \frac{(E_M)\,(R)}{(r_D + R)}$$

$$E_L = \frac{(170)\ (5000)}{3.14\ (29.6 + 5000)}$$

E_L = 54.1 volts (by sliderule)

3. Percent difference between two values of DC voltage

$$\%\ \text{Diff} = \frac{E_L\ (\text{comp}) - E_L\ (\text{meas}) \times 100}{E_L\ (\text{meas})}$$

$$\%\ \text{Diff} = \frac{54.1 - 52.3 \times 100}{52.3}$$

% Diff = 3.45 (by sliderule)

4. DC current

$$I_L = \frac{E_L}{R}$$

$$I_L = \frac{52.3}{5000}$$

I_L = 10.45 mA (by sliderule)

5. DC current

$$I_L = \frac{E_M}{(r_D + R)}$$

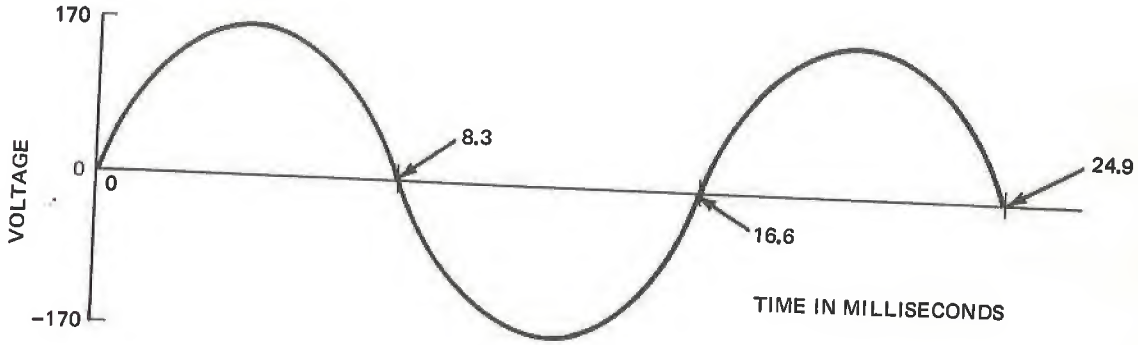
$$I_L = \frac{170}{3.14\ (29.6 + 5000)}$$

I_L = 10.81 mA (by sliderule)

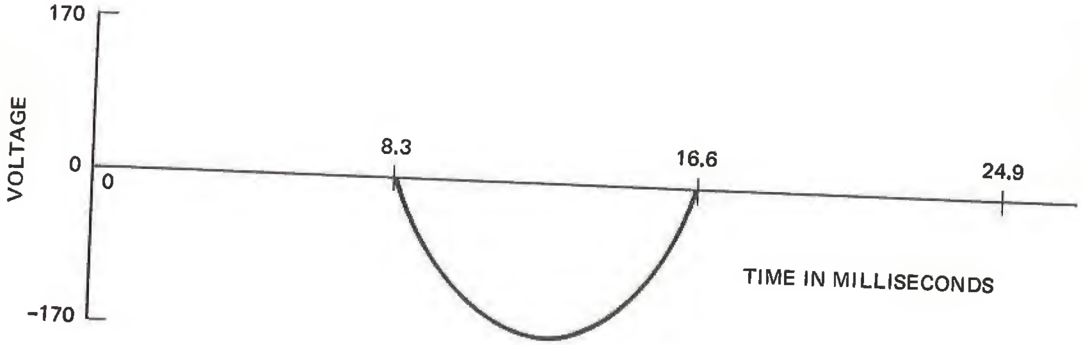
C. **Curves:** See graph sheet attached. (See page 187.)

ANALYSIS OF RESULTS

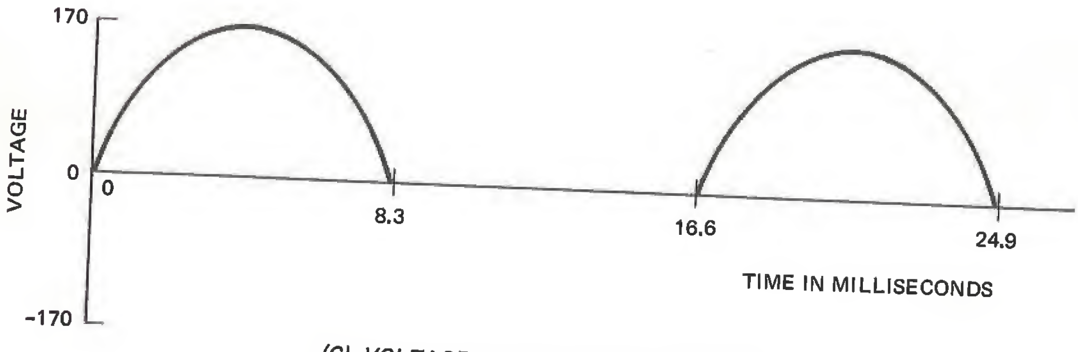
The circuit shown in figure 1 is a simple halfwave rectifier. The waveform shown in (a) of the graph was applied to the input terminals of the circuit. The input waveform appeared to be sinusoidal and had a peak value of about 170 volts. The period of the input waveform was approximately 16.6 milliseconds. These values were obtained by viewing the waveform with an



(A) VOLTAGE ACROSS THE INPUT



(B) VOLTAGE ACROSS THE DIODE



(C) VOLTAGE ACROSS THE 5K RESISTOR

WAVESHAPES FROM THE CIRCUIT

oscillosocpe. Presuming that they are reliable, they suggest that the frequency of the input was:

$$f = \frac{1}{T} = \frac{1}{0.0166} = 60 \text{ Hz}$$

Consequently, the angular velocity would be

$$\omega = 2\pi f = 6.28 \times 60 = 377 \text{ rad/s}$$

As a result, if the waveshape were sinusoidal, it could be represented by the equation,

$$e_i = E_M \text{ Sin } \omega t$$

where E_M = 170 volts and ω = 377 in the experimental circuit.

The forward resistance of the diode was almost surely nonlinear. However, to simplify the analysis, it was assumed to be linear. This assumption is valid only if the forward resistance is a small enough portion of total circuit resistance so that its effect is also small. To insure that this condition did indeed exist in the experimental circuit, the waveshape of the voltage across the load was also viewed. This waveform is shown in (c) of the graph. The peak value of this voltage was found to be approximately 169 volts. The voltage drop across the forward diode resistance (r_D) will be the difference between the peak input voltage (170 volts) and the peak load voltage (169 volts). In other words, the voltage across r_D was about 1 volt. The approximate value of r_D would then be

$$\frac{1}{r_D} = \frac{169}{5000} \text{ or } r_D = 29.6 \text{ ohms.}$$

This value was considered sufficiently small when compared to the load resistance to justify the assumption of linearity.

Diagram (c) of the graph also provides some evidence that the reverse resistance of the diode is very much larger than the load resistance. If this were not the case, the voltage in the interval between 8.3 ms and 16.6 ms in (c) of the graph would not have been so nearly zero. Consequently, it shall be assumed that *no* current flows in the circuit when the input polarity is such as to require reverse flow through the diode.

The waveform of the voltage across the diode was also viewed and it indicated that the results suggested above were valid. This waveform is shown in (b) of the graph. This waveform also helps to establish the validity of the other two, since it is known from Kirchoff's law that the input voltage must equal the sum of the voltage drops within the circuit. Visual inspection of the graph reveals that the graphic sum of (b) and (c) will at least approximate the input voltage waveform, (a).

Under the presumption that the input voltage may be represented by

$$e_i = E_M \text{ Sin } \omega t,$$

it becomes possible to represent the circuit current by the equations

$$i = \frac{E_M}{r_D + R} \text{ Sin } \omega t \text{ if } 0 < \omega t < \pi$$

and

$$i = 0 \text{ if } \pi < \omega t < 2\pi$$

This conclusion is strengthened by the results presented in (c) of the graph.

The DC current that will flow through the load will be the average current taken over the interval $0 < \omega t < 2\pi$. Or,

$$I_L = \frac{1}{2\pi}\int_0^{2\pi} i\,dt = \frac{1}{2\pi}\int_0^{2\pi} \frac{E_M}{r_D + R} \text{ Sin } \omega t\, d\omega t$$

which reduces to

$$I_L = \frac{E_M}{(r_D + R)}$$

This value which resulted from this equation differed from the value measured in the experiment by 3.44%. This difference could have arisen from any of several sources, some of which will be discussed later in this analysis.

Having the equation for the DC current, it is possible to arrive at the following equation for the DC load voltage using Ohm's law:

$$E_L = (I_L)\,(R) = \frac{(E_M)\,(R)}{(r_D + R)}$$

And, again, this value agreed with the measured data to within 3.5%.

The causes underlying the differences between the computed and measured quantities in this experiment could be divided into two categories:

1. Inadequacies on the part of the analysis presented above.
2. Difficulties rising out of the choice and use of the instruments.

The analysis presented may have been in error in that the input waveform may not have been exactly sinusoidal. Also, the assumption of linearity on the part of the forward diode resistance almost certainly introduced a small amount of discrepancy. The effect of non-linearity would have been the alteration of the current waveshape so that it would not have been exactly sinusoidal even if the input voltage were. Assuming the reverse current to be non-existant also introduced some error. It is felt, however, that errors of this type were probably extremely small compared to errors of the second type.

There are several ways in which the instruments used in the experiment could have affected the results. To start with, the oscilloscope was used to measure the peak voltages which were, in turn, used to determine the value of r_D, as well as in the equations for E_L and I_L. The accuracy of these measurements was probably no greater than ± 5%. Also, since the value of r_D depended upon the difference between two virtually equal quantities, a small error in one of these measurements would have had a large effect on the value of r_D. For example, if the peak voltage across the load had been 168 volts instead of 169 volts (a difference of only about 0.6%), the resulting value of r_D would have been 59.6 ohms (a difference of about 100% over the previous value). As dramatic as this variation is, its effect on the final results would not have been great, since even if r_D were 59.6 ohms, it still contributes only about 1% of the total circuit resistance.

Connecting either the oscilloscope or the VTVM across the load, of course, causes some "loading" effect. However, since both of the instruments had input resistances on the order of 10 megohms this effect could not have been great.

Perhaps the greatest source of error in the experiment arose from the accuracy of the instruments. The oscilloscope, VTVM, and resistance decade were all of about 5% accuracy and, in some cases, the instrument errors would have been additive. For example, if both the VTVM and the resistance decade contributed a 5% error in the same direction, the values of E_L and I_L could have been altered by a considerable amount. To illustrate this effect, suppose that the VTVM read 50 volts while the true value was 47.5 volts (meter reads 5% high), and the 5kΩ resistor was actually 5250 ohms (5% too large), then the current would seem to be

$$I_L = \frac{E_L}{R} = \frac{50}{5000} = 10 \text{ mA}$$

while it was actually

$$I_L = \frac{E_L}{R} = \frac{47.5}{5250} = 9.05 \text{ mA}$$

The total error in this case would be close to 10%. If any reading error were added to this value, the total error would be correspondingly higher.

As a result, it would seem that the actual differences encountered in the experiment are well within the limits established by the quality of the instruments, and are thus considered to be acceptable.

In conslusion, it can be said that the circuit encountered in this experiment performs effectively as a simple halfwave rectifier, and that the analysis technique employed is effective in predicting the DC load voltage and current in such a circuit.

PROBLEM SOLUTIONS.

1. Given:

E_i = 117 VRMS r_D = 500 ohms

f = 60 Hz R = 30K ohms

Find:

E_L, I_L

$$E_M = \sqrt{2}\,E_i = 1.414 \times 117 = 166 \text{ volts}$$

$$I_L = \frac{E_M}{(r_D + R)} = \frac{166}{3.14\,(500 + 3000)}$$

$$= \frac{166}{9.15 \times 10^4} = \underline{\underline{1.81 \text{ mA}}}$$

$$E_L = I_L R = 1.81 \times 10^{-3} \times 3 \times 10^4 = \underline{\underline{54.3 \text{ volts}}}$$

2. Since the DC value of the input is zero, the voltage across the diode (E_D) plus the voltage across the load (E_L) must also be zero.

$$E_D + E_L = 0$$

$$E_D = -E_L = \underline{\underline{-54.3 \text{ volts}}}$$

EXPERIMENT 1 ______________ Name ______________________________

Date: ______________________ Class __________ Instructor __________

R_L Ohms	E_1 Volts	I_1 Ma.	E'_1 Volts	E_2 Volts	I_2 Ma.	E'_2 Volts	E_3 Volts	I_3 Ma.	E'_3 Volts	R'_L Ohms

First voltage measurement circuit checked by ______________________________

Instructor

First current measurement circuit checked by ______________________________

Instructor

Fig. 1-5 Data Table

EXPERIMENT 2 ____________ Name ____________

Date: ____________ Class ____________ Instructor ____________

Wire	Length	Dia. (First)	Dia. (Second)	Dia. (Third)	Ave. Dia.	Area Cir. Mils	R Meas.	R Comp	Cross Sect. Area
4 in. No. 30						X			X
6 in. No. 30						X			X
8 in. No. 30									
10 in. No. 30						X			X
12 in. No. 30						X			X
8 in. No. 26									
8 in. No. 28									
8 in. No. 32									
8 in. No. 34									

Fig. 2-4 The Data Table

EXPERIMENT 3

Date: ____________________

Name ______________________________

Class ____________ Instructor ____________________

150 Ohm Resistor				Glo-Bar Resistor				28V Pilot Lamp			
E	I	R_1	r_1	E	I	R_2	r_2	E	I	R_3	r_3

Fig. 3-4 The Data Table

EXPERIMENT 4

Date: ______________________ Name ______________________

Class __________ Instructor ______________________

	Column 1	Column 2	Column 3
E			
I			
R			
P_1			
P_2			
P_3			
P_4			
P_i			

Fig. 4-4 The Data Table

Circuit checked by ______________________
(Instructor)

EXPERIMENT 5

Date: ____________ Name ____________ Class ________ Instructor ____________

LINEAR CIRCUIT VALUES		
Qty	Measured	Computed
E_T		X
I		
E_1		
E_2		
P_T		
P_1		
P_2		
R_1		X
R_2		X
R_T		

NONLINEAR CIRCUIT VALUES		
Qty	Measured	Computed
E'_T		X
I'		
E'_1		
E'_2		

E-I PLOT DATA

E Volts	0	3	6	9	12	15	18	21	24	27	30
I											

Fig. 5-6 The Data Table

EXPERIMENT 6

Name ______________________

Date: ______________________

Class ____________ Instructor ______________________

		E_s = Volts			
R_L	E_L	R_s	P_L	P_T	% eff.
2kΩ					
1.8kΩ					
1.6kΩ					
1.4kΩ					
1.2kΩ					
1.1kΩ					
1.0kΩ					
900Ω					
800Ω					
700Ω					
600Ω					
500Ω					
400Ω					
300Ω					
200Ω					
100Ω					
50Ω					
0					

Fig. 6-2 The Data Table.

EXPERIMENT 7 Name ______________________

Date: ______________ Class ____________ Instructor ______________

Qty	Measured	Computed
R_1		250 ohms
R_2		75 ohms
R_3		150 ohms
R_{12}		
R_T		
E_T		X
I_1		
I_2		
I_3		
I_{23}		
I_T		
I'_1		
I'_2		
I'_T		

Fig. 7-5 The Data Table

EXPERIMENT 8 Name ____________

Date: ____________ Class ____________ Instructor ____________

Qty.	Measured Values	Computed Values
R_1		250 ohms
R_2		100 ohms
R_3		150 ohms
R_4		75 ohms
R_T		
E_T		$\approx$ 30 volts
E_1		
E_2		
E_3		
E_4		
I_1		
I_2		
I_3		
I_4		
P_T		
P_1		
P_2		
P_3		
P_4		

Fig. 8-5 The Data Table

EXPERIMENT 9

Date: ______________ Name ______________

Class ______________ Instructor ______________

Qty.	Measured Value	Computed Value
E_A	30 volts	30 volts
E_B	24 volts	24 volts
R_1		100 ohms
R_2		150 ohms
R_3		250 ohms
R_4		75 ohms
R_A		
R_B		
I_1		
I_2		
I_3		
I_4		
E_1		
E_2		
E_3		
E_4		
P_1		
P_2		
P_3		
P_4		
P_A		
P_B		
P_T		

Fig. 9-4 The Data Table.

EXPERIMENT 10 ______________ Name ______________________

Date: ______________________ Class __________ Instructor ______________

Qty	R_1	R_2	R_L	E_A	E_B	I_L	E_L	E_s
Computed	150	75	100	25V	15V			
Measured								

	Thévenized Circuit			Nortonized Circuit			Superposition		
Qty	R_s	E_L	I_L	I_s	E_L	I_L	I_{LA}	I_{LB}	I_L
Computed									
Measured									

Fig. 10-11 The Data Table

EXPERIMENT 11

Date: ____________________ Name ____________________

Class __________ Instructor ____________________

Qty	Measured Values	Computed Values	Percent Difference
R_1		75Ω	
R_2		150Ω	
R_3		250Ω	
R_4		100Ω	
E_T		15V	
I_L			
I_1			
I_2			
I_3			
I_4			
I_T			

Data for Load Voltage Plot		
R_4 Ohms	E_s	E_L
100		
200		
300		
400		
500		
600		
700		
800		
900		
1000		

Fig. 11-6 The Data Table.

EXPERIMENT 12

Date: ____________________ Name ____________________

Class __________ Instructor ____________________

Ammeter Data

I_T (mA)	I_m
1.0	
2.0	
3.0	
4.0	
5.0	
6.0	
7.0	
8.0	
9.0	
10.0	

Voltmeter Data

E_T (volts)	I_m	E_m
1.0		
2.0		
3.0		
4.0		
5.0		
6.0		
7.0		
8.0		
9.0		
10.0		

Ohmmeter Data

R_x (ohms)	R_I
0	
100Ω	
500Ω	
1kΩ	
10kΩ	
20kΩ	
40kΩ	
40kΩ	
60kΩ	
100kΩ	

R_m (Meas)	R_s (Comp)	ℓ_s (Meas)	R_s (exp)	R_v (exp)	R_v (comp)	E_b (Meas)	E_b (Comp)

Fig. 12-10 The Data Table

EXPERIMENT 13

Date: ______________ Name ______________

Class ______________ Instructor ______________

R_V Comp.	E_T Meas	R_1 Meas	R_2 Meas	I_T Comp	E_1 Comp	E_2 Comp	S_1 Comp

Fig. 13-5a Circuit Data

Qty.	Multimeter I & Const. V meter	Multimeter I & VOM V	0-1 mA I & Multimeter V
I_T			
I_1			
E_1			
I_1'			
E_1'			
E_1''			
I_2			
E_2			
I_2'			
E_2'			
E_2''			

Fig. 13-5b Instrument Comparison Data.

EXPERIMENT 14

Date: ____________ Name ____________

Class ________ Instructor ____________

I	T	Q_T	E	C_1

Part 1

I	T	Q_T	E	C_2

Part 2

I	T	Q_T	E_1	E_2	Q_1	Q_2	Q'_T	% Diff Q_T	C_T	C'_T	% Diff C_T

Part 3

I	T	Q_T	E_1	E_2	Q_1	Q_2	Q'_T	% Diff Q_T	C_T	C'_T	% Diff C_T

Part 4

Fig. 14-5 The Data Table

EXPERIMENT 15 Name ______________

Date: ______________ Class ________ Instructor ______________

RC	E_o	E_c	I_o	Q	C	% Diff E
	12.0V					

Part 1

T (Min)	E_R (VOM)	I	E_c
0			
0.5			
1.0			
1.5			
2.0			
2.5			
3.0			
3.5			
4.0			
4.5			
5.0			
5.5			
6.0			
6.5			
7.0			
7.5			
8.0			
8.5			
9.0			
9.5			
10.0			

Part 3

T (Min)	E_c
0	
0.5	
1.0	
1.5	
2.0	
2.5	
3.0	
3.5	
4.0	
4.5	
5.0	
5.5	
6.0	
6.5	
7.0	
7.5	
8.0	
8.5	
9.0	
9.5	
10.0	

Part 2

Fig. 15-7 The Data Table

EXPERIMENT 16 ______________ Name ______________________

Date: ______________________ Class ____________ Instructor ______________

Gen Freq.	Gen Output Level	E_{p-p} (Scope)	T (Scope)	f (Scope)	E_{p-p} (VTVM)	% Diff E_{p-p}	% Diff (f)
60 Hz	20%						
100 Hz	30%						
120 Hz	40%						
150 Hz	50%						
200 Hz	60%						
250 Hz	70%						
400 Hz	80%						
600 Hz	100%						

Fig. 16-4 The Data Table.

EXPERIMENT 17 ______________ Name ______________

Date: ______________ Class ______________ Instructor ______________

E (VTVM)	R	$E_{p\text{-}p}$ (scope)	E_m (scope)	E_m (VTVM)	% Diff E_m	I (meas)	I (comp)
36V							
32V							
28V							
24V							
20 V							

E (VTVM)	% Diff I	P (meas)	P (comp)	% Diff P	P_m (wattmeter)	P_m (comp)	% Diff P_m
36V							
32V							
28V							
24V							
20V							

Fig. 17-5 Data Table

EXPERIMENT 18 ______ Name ______

Date: ______ Class ______ Instructor ______

I_1	E_1	L_1	I_2	E_2	L_2	I_s	E_s	L_s	I'_s	E'_s	L'_s	I_p	E_p	L_p

Part 1 (No Coupling)

I_1	E_1	L_1	I_2	E_2	L_2	I_s	E_s	L_s	I'_s	E'_s	L'_s

Part 2 (Coupled)

L_s (Comp)	% Diff L_s, L_s	% Diff L_s, L'_s	L_p (Comp)	% Diff L_p, L_p

Part 3 (No Coupling)

M	L_s (Comp)	% Diff L_s, L_s	L'_s (Comp)	% Diff L_s, L'_s

Part 4 (Coupled)

Fig. 18-7 The Data Table

EXPERIMENT 19 ______ Name ______

Date: ______ Class ______ Instructor ______

E_I	Hz	E_C	E_R	I	X_C (meas)	X_C (comp)	% Diff X_C	y	E_y	Sin θ	θ
6V	25										
6V	50										
6V	100										
6V	200										
6V	300										
6V	400										
6V	500										
6V	600										
6V	700										
6V	800										
6V	900										
6V	1000										

E_I	Hz	E_L	E_R	I	X_L (meas)	X_L (comp)	% Diff X_L	y	E_y	Sin θ	θ
6V	25										
6V	50										
6V	100										
6V	200										
6V	300										
6V	400										
6V	500										
6V	600										
6V	700										
6V	800										
6V	900										
6V	1000										

Fig. 19-8 The Data Table

EXPERIMENT 20 ______________ Name ______________________________

Date: ______________________ Class ____________ Instructor ______________

RL Circuit Data				E_R =			R_C =		
f Hz	E_T	I	Z Meas	θ Meas	X_L Comp	Z Comp	θ Comp	% Diff Z	% Diff θ
50									
100									
150									
200									
250									
300									
400									
500									
600									

Fig. 20-12 The Data Tables

RC Circuit Data				E_R =					
f Hz	E_T	I	Z Meas	θ Meas	X_C Comp	Z Comp	θ Comp	% Diff Z	% Diff θ
50									
100									
150									
200									
250									
300									
400									
500									
600									

RLC Circuit Data				E_R =				R_C =		
f Hz	E_T	I	Z Meas	θ Meas	X_L Comp	X_C Comp	Z Comp	θ Comp	% Diff Z	% Diff θ
50										
100										
150										
200										
300										
400										
500										
600										

Fig. 20-12 Data Tables (continued)

EXPERIMENT 21 ______________ Name ______________________________

Date: ______________________ Class ____________ Instructor ______________

Data from Inductor Circuit						R_C =		
f Hz	I	E_L	Z Meas.	θ Meas.	Z Comp.	θ Comp.	% Diff Z	% Diff θ
10								
20								
30								
40								
50								
60								
70								
80								
90								
100								

Fig. 21-4 The Data Table

Data from the RC Circuit								
f Hz	I	E	Z Meas.	θ Meas.	Z Comp.	θ Comp.	% Diff Z	% Diff θ
10								
20								
30								
40								
50								
60								
70								
80								
90								
100								

Data from the Parallel Combination								
f Hz	I	E	Z Meas.	θ Meas.	Z Comp.	θ Comp.	% Diff Z	% Diff θ
10								
20								
30								
40								
50								
60								
70								
80								
90								
100								

Fig. 21-4 The Data Tables (continued)

EXPERIMENT 22 ______________ Name ______________

Date: ______________ Class ______________ Instructor ______________

Quantity	Value
E_T	
I_T	
E_R	
E_C	
P_{ave}	
P_R	
P_C	
θ	
P_{ave}	
P_{app}	
P_X	
pf	
% Diff P_{ave}	
I_T Comp.	
% Diff I_T	
P_{app} Comp.	
% Diff P_{app}	

Fig. 22-8 The Data Table

EXPERIMENT 23 ______________ Name ______________________

Date: ______________________ Class __________ Instructor ______________

Qty	Value
R_C	
E_T	
f_o meas.	
E_C meas.	
I_O	
R_T	
E_R	
X_L	
Q_o	
E_C comp.	
% Diff E_R, E_T	
% Diff E_C	
f_o comp.	
% Diff f_o	
BW meas.	
BW comp.	
% Diff BW	

f (Hz)	E_1	I_T	Z_T	f (Hz)	E_1	I_T	Z_T
50				155			
60				160			
70				165			
80				170			
90				175			
100				180			
105				185			
110				190			
115				195			
120				200			
125				210			
130				220			
135				230			
140				240			
145				250			
150				260			

Fig. 23-8 The Data Tables

EXPERIMENT 24 ______________ Name ______________

Date: ______________ Class ______________ Instructor ______________

Qty	Value
R_c	
f_m	
I_o	
E_c	
f_o	
f_o'	
BW meas	
Q	
BW comp	
% Diff BW	
f_o	
i_c	
X_c	
i_c	
% Diff i_c	
R_T (meas)	
R_T (comp)	
% Diff R_T	

f Hz	E_1	I_T	Z_T
50			
60			
70			
80			
90			
100			
105			
110			
115			
120			
125			
130			
135			
140			
145			
150			

f	E_1	I_T	Z_T
155			
160			
165			
170			
175			
180			
185			
190			
195			
200			
210			
220			
230			
240			
250			
260			

Fig. 24-7 The Data Tables

EXPERIMENT 25 ____________ Name ____________

Date: ____________ Class ____________ Instructor ____________

Resistance Decade Only

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(a)

Decade and 12 mfd cap

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(b)

Source Impedance Data

R_c	R_s	X_s	Z_s

Fig. 25-7 The Data Tables

Decade and 11 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(c)

Decade and 10 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(d)

Decade and 7 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(e)

Fig. 25-7 The Data Tables (continued)

EXPERIMENT 25 ____________ Name ____________________

Date: ____________________ Class ____________ Instructor ____________

Decade and 6 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(f)

Decade and 5 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(g)

Decade and 2 mfd cap.

R_L Ohms	E_1	I_L	P_L	Z_L
0				
50				
100				
150				
200				
250				
300				
350				
400				
450				
500				
550				
600				
650				

(h)

Fig. 25-7 The Data Tables (continued)

EXPERIMENT 26 ________________ Name ________________

Date: ________________ Class ____________ Instructor ____________

No Load

Qty	Value
R_p	
R_s	
E_1	
E_s	
I_1	
N_p/N_s	
Z_p	
X_p	
θ_p	

100-ohm Load

Qty	Value
E_p	
I_p	
E_s	
I_s	
I_p'	
N_p/N_s	
% Diff N_p/N_s	
Z_i	
Z_R	
% Diff Z_i, Z_R	

L & C Load

Qty	Value
f_1	
f_2	
f_3	

Fig. 26-5 The Data Tables

EXPERIMENT 27 ______ Name ______

Date: ______ Class ______ Instructor ______

Turns Ratio Data

V_p	V_{s1}	V_{ct}	N_1/N_2	$N_1/(N_2 + N_3)$	N_2/N_3	N_3/N_1

Load Power Data

oad esistance ${}_L/I_L$ (ohms)	Turns ratio =		Turns ratio =		Turns ratio =		Turns ratio =	
	V_L	$V_L I_L$	V_L	$V_L I_L$	V_L	$V_L I_L$	V_L	$V_L I_L$
0								
50								
100								
150								
200								
300								
400								
500								
600								
700								
800								
900								
1000								
1200								
1400								
1600								
1800								
2000								
2400								
2800								
3200								
3600								
4000								
4500								
5000								

Fig. 27-6 The Data Table.